

THE
PRACTICAL FIGURER;
OR, AN
IMPROVED SYSTEM
OF
ARITHMETIC.

CONTAINING,

A large Exemplification of the several RULES in common ARITHMETIC, with the Doctrine of Vulgar and Decimal Fractions; abounding with numerous Examples, useful, subtle, and entertaining, for gaining the Attention, and improving the Genius of Youth.

Many new and concise Methods for finding the Price of Goods, Interest of Money, &c. with one Rule entirely New, called IRREGULAR PROGRESSION, never before heard of in ARITHMETIC.

A large practical System of ALGEBRA, containing a great variety of new and entertaining Problems, with a direct Method of solving CUBIC EQUATIONS, by an easy and simple Operation.

By WILLIAM HALBERT,
SCHOOLMASTER AT AUCHINLECK. *R*

Omne tulit punctum, qui miscuit utile dulci.

Hos.

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MDCCLXXXIX.

ENTERED IN STATIONERS HALL.



TO
THE RIGHT HONOURABLE,
AND
HONOURABLE,
THE NOBILITY AND GENTRY,
THE REVEREND CLERGY,
AND
RESPECTABLE LAITY,
ENCOURAGERS OF THIS WORK,
THE FOLLOWING ESSAY
IS MOST RESPECTFULLY DEDICATED,

By their much obliged,
And very humble servant,

THE AUTHOR.

P R E F A C E.

ARITHMETIC is the art of reckoning, computing and calculating by number, the method of managing, and casting up of accounts by figures.

As the subject of this science is number, so the purposes it serves, and the ends to which it is applied are *numerous*, its advantages many, and its uses extensive. For as number is connected with every thing we do, or are conversant about, so an acquaintance with the art of numbering, is on that account highly necessary to people of every calling and denomination, whether men of letters or business. The merchant and mechanic, the landholder and sea-trader, the lawyer and warrior, the physician and philosopher, and even the divine derive advantage from it: But especially in every practical part of the mathematics, and science depending upon them, in navigation and architecture, in geography and astronomy, and other parts of natural philosophy, those amusing and interesting studies, the knowledge of figuring is an indispensable requisite. Arithmetic may therefore be considered as an introduction to these sciences, and studies of importance, without which they are unattainable

tainable, and impracticable, and without the aid of which, many valuable discoveries, in some of them, had never been made.

This art, viewed in so extensive a regard, displays a wide field wherein to exercise the faculties of youth, and were we to set out with full rein on this course, it might be doubtful when we should reach the goal; were we to be proficient in every branch of the art of figuring, and masters of every science with which it is intimately connected. But what commonly writers on this subject consider as arithmetic, comprehends only the reduction of coins, finding the price of goods, interest of money, or whatever belongs to bargaining among men of business, and perhaps something of mensuration; but this rather requires a treatise by itself, at least, the intended size of the present volume, does not admit entering upon that part of the subject.

As the universal usefulness of arithmetic is conspicuous, so its antiquity must undoubtedly be very great; for wherever trade and merchandize took place, in any considerable degree, something of that art must soon naturally have followed; on this account, the Arabians are said to have been the first inventors of it, and the ingenious in succeeding ages, through every civilized and trading nation, have gradually improved, and made new discoveries in it; perhaps

as much in our own country as any other, where it must also be of considerable long standing, for in some very old authors, still extant, we find great advances made, not only in the useful, but also in the curious parts of the science; and which, though far short of the improvements of later ages, and the perfection it is now arrived at, yet sufficiently shews it, even then, to have been long past its infancy.

The early writers on this subject, well knowing the ignorance of the world, and the confined progress of learning in their times, were obliged to treat minutely on the first rudiments of the science, and to be very full, and particular, on every small, and most trifling circumstance, conforming to the capacity of their readers; and they were to be commended in thus adapting their instructions to such as were destitute of other helps; teachers, at that time, very probably, being rarely to be met with, except in great cities, and the most populous parts of a country. But now that the art is become so common, as in a great measure to be understood by the generality of mankind, and when there are few, even of the vulgar, but who know something of it, I think modern writers might spare filling up, three parts in four of their work with *school boys prattle*, and such weak, and shallow assertions, as are ridiculous to be put in print. It is now superfluous to demonstrate that

two and two make four, that an even number can be divided into two equal parts, or that the difference of two numbers, added to the lesser, will make the greater ; however such remarks might be excuseable, or necessary about the time of the reformation, they are now insipid, and considered as lumber, being suited only to mere children, who cannot be supposed to ponder upon books, and to such as have made any proficiency, they are too insignificant to bear a repetition. Perhaps in former ages, men were come to years of discretion, before they entered upon this branch of learning, in which case, such books might be more serviceable than in our times, when they are boys who apply to it ; these never learn the first principles from books, but by oral instruction, and imitation of others, and must have something of practice before demonstrations, without which they can never be understood ; and the reasons of things make way into their mind by frequent examples, and are best retained when thus acquired ; and there are many, whom I have known make good proficiency in such a manner, on whom no reasoning could have any effect, at least till after they had made considerable advances. On these considerations, in the following system, I omit the long illustrations, and hackneyed remarks on the common rules, which take up no small part of other performances, and thereby save unnecessary

unnecessary labour, and the Reader from paying for waste paper, from which no profit can be reaped, and so save room to furnish him with something more substantial, and of greater utility, by a careful perusal of which, he may reasonably expect some benefit.

It is alledged by some, that this subject is now exhausted, and capable of no further improvement, as several modern productions seem to be little else than transcripts of one another, or of their predecessors in the same line; but the contrary will be evinced in the present performance, where there is both something new, and something done in a manner different from all preceding authors, and such an assemblage of questions and examples as never appeared before: And these not picked out of other publications, but newly coined from the mint, (no doubt there are imitations, which could not be avoided,) and such as will exercise the faculties, and entertain the fancy at the same time, which I find to have a good effect in the instructing of youth; for when no questions are proposed, but such as are common in real business, a learner is apt to weary of them, being more delighted with something that is diverting, and which never fails, in this case, to be instructive also; for when a boy has been accustomed to resolve puzzling and amusing questions, such as are connected with the real occurrences of life

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become

become entirely easy to him, and he can discuss them instantaneously.

Here too a common objection will be answered, or prevented, That there are not sufficient examples in any one book, for a scholar's exercise, so as to make the practice of figuring familiar to him, but that teachers are obliged to use many books, picking and chusing from each, something to answer their purpose, and so, with great pains, patch up some tolerable form, or set of Arithmetic. This labour may be saved here, where examples are certainly numerous enough for the practice of any learner, (as I imagine) though I own, where time and circumstances allow, the perusing of many books is not to be discommended; for something may be gathered with more facility from one than another, and by the manner of expression, one may throw a light upon another.

I might, no doubt, according to common custom, have demonstrated, that of four proportional numbers, *the product of the extremes, is equal to that of the means*, and that consequently, *the product of the means, divided by the first term, quotes the fourth proportional*, and hence the operation of the Rule of Three, where three numbers are given, to find a fourth in proportion: But this, as many other topics, has been so often insisted upon, that it is become a thread-bare subject, and though not so trifling, is going into a contrary

trary extreme, no less foreign to the purpose, being a geometrical way of reasoning; for upon arithmetical principles, the Rule of Three may be accounted for in the most simple and intelligible manner, by the *living voice*, so as to be easily understood; whereas, to harangue upon *similitudes of Ratios*, and *mutual habitudes*, would not serve to inform, but to stupify children learning the first principles of Arithmetic; and ought therefore altogether to be omitted till a boy has perused something of the Elements of Geometry, and his judgment, and other faculties, are become mature, and susceptible of such demonstrations.

When an amusing or intricate question is proposed, I generally give a direction how it may be resolved, and which, though concise, will be sufficient to such as have properly exercised themselves in less perplexed examples, and by degrees have accustomed themselves to a close way of thinking on the subject: And lest some of those that are not illustrated in this manner, should prove hard for a learner, or even some teachers, I have selected those I thought any how difficult, and given a short exposition of them, in an Appendix, subjoined to the arithmetical part, with references by the chapter, and number of the example: In this place also I insert a long train of what I call *Questions at Random*, to which I give only the answers; for these having occurred, after the manuscript was

was finished, are to be considered as supernumerary to the book: They will serve to entertain and amuse the learner, and the more, that they are placed without order, or regard had to what rule they belong.

In the second department of the work, I have conformed more to common practice, and have said some things adapted to the capacities of children, as Algebra is a thing not yet altogether so commonly known, at least so far as my acquaintance reaches; however here also I studied brevity, insisting chiefly on the practical part, and giving a multiplicity of examples, which I always think throw a more striking light upon the subject than precept.

As to my new Rule of *Irregular Progression*, I shall not dispute for the propriety of the name: it was the first that occurred. There are many, and numberless kinds of these progressions, and it might require a volume, to expatiate upon them at large; many curious discoveries, I am certain, may be made in them, and of these I have given several instances, which may be pretty easily comprehended, as set forth here, but which cost me inexpressible pains, and expence of time and labour; one thing I had in view, (which was the applying them to the resolution of the *higher adjected Equations*) I have not been able to accomplish; and shall leave the further prosecution of them to those of more sagacity, and fewer avocations than I have at present.

Upon

Upon the whole, I think this book will be found to be original, which is a novelty, and gives a value to things of itself, though otherwise insignificant and trifling, which I am unwilling to believe this performance to be; and am not without hopes, that, after perusal, my Readers will, in some measure, concur with me in their sentiments concerning it, and whatever deficiencies or mistakes may be discovered, I shall beg the indulgence of the considerate and ingenuous to apologize for me. Allowances for escapes and oversights must be made to every man, and the most skilful will be readiest to make them. We all know, they are often most forward to find fault, who know least how to amend.

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TABLES of Money, Weight, Measure, &c.

English Money. 4 farthings = 1 penny, 12 pence = 1 shilling, and 20 shillings = 1 pound.

Troy Weight. 24 grains = 1 penny-weight, 20 dwt. = 1 ounce, and 12 oz. = 1 pound.

Apothecaries Weight. 20 grains = 1 scruple, 3 scruples = 1 dram, 8 drams = 1 ounce, 12 oz. = 1 pound.

Avoirdupois Weight the greater. 28 pounds = 1 quarter, 4 quarters = 1 hundred-weight, 20 Cwt. = 1 ton.

Avoirdupois Weight the lesser. 16 drams = 1 ounce, 16 ounces = 1 pound, 14 lb. = 1 stone.

Wool Weight. 7 lb. = 1 clove, 2 cloves = 1 stone, 2 stones = 1 todd, $6\frac{1}{2}$ todods = 1 weigh, 2 weighs = 1 sack, 12 sacks = 1 last.

Dutch Weight. 16 drops = 1 oz. 16 oz. = 1 lb. 16 lb. = 1 stone.

Scotch Iron Weight. The same as the Dutch; but 24 oz. in the pound in this place, though different in other counties.

Cloth Measure. 4 nails = 1 quarter, 4 qrs. = 1 yard, English; or, 4 nails = 1 qr. 4 qrs. = 1 ell Scotch.

Dry Measure, English. 8 pints = 1 gallon, 2 gallons = 1 peck, 4 pecks = 1 bushel, 4 bushels = 1 coomb, 2 coombs = 1 quarter, 5 qrs. = 1 wey, 2 weys = 1 last.

Liquid Measure. 231 solid inches = 1 gallon, 63 gallons = 1 hoghead, 2 hhds. = 1 pipe, 2 pipes = 1 tun, wine measure; and 282 inches = 1 gallon, $8\frac{1}{2}$ gallons = 1 firkin, 2 firkins = 1 kilderkin, 2 kilderkins = 1 barrel, beer measure.

Dry Measure, Scotch, 4 lippies = 1 peck, 4 pecks = 1 firiot, 4 firlots = 1 boll, 16 bolls = 1 chaldier.

Liquid

Liquid Measure. 4 gills = 1 mutchkin, 2 mutchkins = 1 chopin, 2 chopins = 1 pint, 2 pints = 1 quart, 2 quarts = 1 gallon.

Long Measure. 3 barley-corns = 1 inch, 12 inches = 1 foot, 3 feet = 1 yard, $5\frac{1}{2}$ yds. = 1 pole, 40 poles = 1 furlong, 8 furlongs = 1 mile.

Land Measure. $30\frac{1}{2}$ yards a pole, 40 poles = 1 rood, 4 roods = 1 acre, English: Or 36 ells = 1 fall, 40 falls = 1 rood, 4 roods = 1 acre, Scotch.

Time. 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 7 days = 1 week, 4 weeks = 1 month, 13 months = 1 year, nearly.

Explanation of Characters.

This +, called *plus*, is the sign of addition, and may be read *and*. This —, called *minus*, is the sign of subtraction, and may be read *wanting*. The sign of multiplication is ×, and of division ÷, and this = is called the mark of *equality*. The marks of proportion are two points, thus :, or four, thus ::; and when four proportional numbers are stated may be read as follows: 4 : 6 :: 8 : 12; 4 is to 6 as 8 to 12.

CHAPTER I.

An Exemplification of the Common Rules.

THE NUMERATION TABLE.

4	Hund. of thous. of bill.
6	Tens of thous. of bill.
2	Thous. of bill.
3	Hund. of bill.
7	Tens of billions.
4	Billions.
8	Hund. of thous. of mill.
2	Tens of thous. of mill.
6	Thous. of mill.
7	Hundred of mill.
3	Tens of millions.
5	Millions.
4	Hundreds of thousands.
7	Tens of thousands.
8	Thousands.
3	Hundreds.
2	Tens.
1	Units.

If we have occasion to read a longer line than the above, we can say, Trillions, Quadrillions, Quintillions, Sextillions, &c.

§ 2. ADDITION of one Denomination.

246862	376248
728643	279623
256748	516742
27679	567823
84262	679842
3746	237640
2876	123274
1350816	

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VAR. 2.

VAR. 2. *Addition of English Money.*

L. 24	10	6 $\frac{1}{2}$	L. 34	14	2
36	11	7 $\frac{1}{4}$	96	18	4 $\frac{1}{2}$
42	12	8 $\frac{1}{4}$	125	7	6
52	13	7	45	12	1 $\frac{1}{2}$
64	17	9 $\frac{1}{2}$	68	19	11
72	18	6 $\frac{1}{2}$	56	15	3
294	4	9 $\frac{1}{2}$			

A Bill of Parcels.

Mr. John Silver bought of William Chapman,

70 yards broad cloth, at 15s.	L. 52	10	—
80 yards velvet, at 18s.	72	—	—
90 yards fustian, at 4s. 3d.	19	2	6
60 doz. buttons, at 1s. 3d.	3	15	—
125 hanks thread, at 7 $\frac{1}{2}$ d.	4	—	8 $\frac{1}{2}$

Received in full, 151 8 2 $\frac{1}{2}$

VAR. 3. *Addition of Troy Weight.*

lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
24	10	12	14	35	11	12	13
36	9	18	20	28	10	14	21
42	11	16	18	39	6	11	12
94	10	15	11	45	3	17	19
72	10	12	20	25	4	18	22
56	11	13	15	67	8	19	20
42	8	16	17	41	7	18	10
61	10	15	18	24	5	13	14
434	1	2	13				

VAR. 4.

THE COMMON RULES.

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VAR. 4. Addition of Avoirdupois Greater Weight.

Tons.	C.	qrs.	lb.
64	18	3	24
23	14	2	16
37	16	2	17
94	12	3	12
78	17	1	21
41	15	2	15

Tons.	C.	qrs.	lb.
74	18	1	14
36	14	2	15
17	16	2	17
28	17	3	15
62	11	3	10
79	12	2	18

341 16 — 21

VAR. 5. The Lesser Avoirdupois Weight.

St.	lb.	oz.	dr.
64	13	15	15
58	12	13	14
74	11	12	13
56	10	12	14
37	9	13	14
74	10	12	10

St.	lb.	oz.	dr.
24	12	10	12
28	13	14	15
73	10	11	12
92	6	10	14
56	8	9	10
67	9	8	6

368 — 2 —

VAR. 6. Dutch Weight.

St.	lb.	oz.	dr.
64	13	14	15
24	10	14	10
67	12	13	14
45	9	10	12
82	10	12	14

St.	lb.	oz.	dr.
78	12	14	11
38	14	11	10
45	15	13	14
28	10	12	10
37	14	12	13

32 3 4 1

96 11 13 14

§ 2. SUB.

§ 2. SUBTRACTION of Integers.

Minuend	462478	34672438
Subtrahend	264738	24368234
Remainder	197740	

Examples of English Money.

Borrowed	L. 245	10	6	L. 364	18	9 $\frac{1}{2}$
Paid	187	15	6	275	11	2
Unpaid	57	15	—			

Troy Weight.

	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
Bought	26	9	14	16	36	5	12	11
Sold	14	10	17	20	21	7	18	16
Unfold	11	10	16	20				

Avoirdupois Weight.

	Tons.	C.	qrs.	lb.	Tons.	C.	qrs.	lb.
Bought	125	12	1	14	176	13	1	10
Sold	96	18	2	20	89	15	3	12
Unfold	28	13	2	22				

Dutch Weight.

	St.	lb.	oz.	dr.	St.	lb.	oz.	dr.
Bought	763	10	11	14	385	10	12	15
Sold	694	13	14	15	258	11	13	7
Unfold	68	12	12	15				

Tren

Tren Weight.

	St.	lb.	oz.	dr.	St.	lb.	oz.	dr.
Bought	725	14	20	10	700	—	—	—
Sold	684	15	22	13	638	12	14	10
Unfold	40	14	21	13				

Subtraction is proved by adding the remainder to the subtrahend, and the sum is equal to the minuend.

§ 3. OF MULTIPLICATION.

Multiplicand	246728	376425
Multiplier	46	234
	1480368	1505700
	986912	1129275
Product	11349488	752850
		88083450
Multiplicand	2357984	6234567
Multiplier	2468	35791
Product	5819504512	223141387497

CONTRACTIONS.

Any number is multiplied by 10, 100, 1000, &c. by annexing 1, 2, or 3 ciphers to it.

To multiply by a repeating 9, annex a cipher for every 9, and subtract the multiplicand. Thus 2468 by 999, or 9999, may be multiplied thus:

	2468000	24680000
	2468	2468
Product	2465532	24677532

If

THE COMMON RULES.

If you divide any of these products by 9, you have the product by a repeating 1, and this, multiplied by any figure, will give the product by that repeater. Multiply 67854 by 666.

67854000

67854

9)67786146 prod. by 999.

7531794 prod. by 111.

6

45190764 prod. by 666

or thus: 9)67854000

7539333

7539

7531794

6

45190764

When one part of your multiplier is an even part of another, you multiply first by that part, and its product by the other component part. Thus, to multiply by 426, take the product by 6, and multiply it by 71 or, to multiply by 6432168 you double every product, taking care to shift the figures properly.

2456

426

14736

103152

1046256

1234567

6432168

9876536

19753072

39506144

79012288

7940942351256

Multiplication of English Money.

What cost 48 stoncs of wool, at 5s. 9d. per stone?

L. — 5 9

6

1 14 6

8

13 16 —

Or thus: L. — 5 9

8

2 6 —

6

13 16 —

What

THE COMMON RULES.

23

What cost 125 lb. raisins, at 7d. per lb.?

Or thus: 7×5

$$\begin{array}{r} 7 \\ 5 \\ \hline 2 \quad 11 \\ 5 \\ \hline 14 \quad 7 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 3 \quad 10 \times 2 \\ 10 \\ \hline 2 \quad 18 \quad 4 \\ 11 \quad 8 \\ 2 \quad 11 \\ \hline \end{array}$$

L. 3 12 11

L. 3 12 11

What cost 4567 yards of cloth, at 15s. 4d. per yard? Answ. L. 3501 7 4

What cost 500 stones of Hay, at $4\frac{1}{2}$ d. per stone? Answ. L. 9 7 6.

What cost 900 tunns of oil, at L. 45 11 per tun? Answ. L. 22995

§ 4. DIVISION.

Dividend.

Divisor 48)2345678(48869 Quotient.

192.....

425
384

416
384

327
288

398
384

14 Remainder.

2345678 Proof, by adding the remainder,

der, and all the products, missing the dividu-als. Division is also proved by multiplying the quotient by the divisor.

Examples to be wrought.

Dividend. Quotient.
 Divisor 2468)724386756(293511
 Rem. (498461)

Dividend. Quotient.
 Divisor 2864376)12345678901234(4310076
 Rem. (648658)

Dividend. Quotient.
 Divisor 674869286)128673274868456(
 Remainder (672191838)

CONTRACTIONS.

You divide by a single figure without setting down any thing but the quotient. Thus,

6)24680

Quotient 4113—2 remainder.

To divide any number by 10 or 100, cut off a figure or two from the right hand; thus 2468 divided by 100 is 24, and 68 the remainder.

To divide by a repeating 1, subtract the quotient figure from every figure of the dividu- al. And if this is divided again by any figure, it gives the quotient by a repetend of that figure. Thus divide 468246 by 666.

6)
 111)468246(4218 quote by 111

242 .. —

204 703

936

48

To

To divide by any number of 9's, dispose of your number into periods. Set the first period under the second and sum of both under the third, &c. whatever figure is to carry set it in its proper place, and under the right hand set such a figure as would have come, if the work had gone further, then add all as they stand, and cut off as many for your remainder as there were of 9's.

Divide 678,901,234 by 999 or 7755,3322,11 by [9999]

$$\begin{array}{r} 678,901,234 \\ \underline{1 \quad 1} \\ 679580|814 \end{array}$$

$$\begin{array}{r} 7755,3322,1100 \\ 7755,1077 \\ \underline{1 \quad 1} \end{array}$$

$$775610|7821$$

To the last number we added two ciphers to make a complete period, but these places are left blank when we add.

A few Questions to exercise the Learner in these Rules,

Bought a parcel of heavy goods, for which paid in hand L. 125 10, gave bill for L. 96, expended at making the bargain 15s. 3d. paid the expence of packing and cordage L. 2 7 4 $\frac{1}{2}$, and carriage home L. 5 12 6: For how much must I sell these goods, so as to gain L. 100? Answ. L. 330 5 1 $\frac{1}{2}$.

2. The first distribution of prize-money at the Savannah, October, 1762, was said to be thus: Commander in chief L. 86030 17 2; Admiral the same sum; General Elliot L. 17206 3 5; Commodore the same sum; the army's share L. 154855 10 11, and the navy's the same sum: Required the sum total of this distribution? Answ. L. 516185 3.

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3. The

3. The battle of Bannockburn was fought in 1314; how long is it since?

4. How long since playing cards were first used in Europe, which is said to have been about 1390?

5. My eldest son was born when I was 34 years 7 months old, and now when I am 70 years old and a half, what is the age of that son?

Answ. 45 yrs. 11 mon.

6. How many syllables are there in the 119 Psalm?

Answ. 3696.

7. How many men were in a Roman legion, which consisted of 3 manipuli, a manipulus 2 centuriae of 100 men each?

Answ. 6000.

8. How many trees in a forest of 4 miles broad, and 6 long, when the rows every way are 6 ells asunder?

Answ. 2460801.

9. A fruit-woman being asked how many apples she had in her basket, said, That when she told them over 2 by 2 there was one odd, also 3 by 3, 4 by 4, 5 by 5, or 6 by 6, still one was odd; but when she told them over by 7's none was odd: Required the number?

Answ. 721.

10. There are said to be 800 hackney coaches in London, what are the number of wheels and spokes, horse-shoes and nails, two horses being in each?

Answ. 3200 wheels, 38400 spokes, 6400 shoes, 51200 nails.

11. Divide 500000 Spanish dollars

'Mong 20 male, and 30 female scholars;

And in the distribution, let the men

Exceed the women each threescore and ten.

Answ. A male 10042, a female 9972.

12. Sixty men, 50 women, 24 boys, and 30 girls being at a penny-wedding, are to pay L. 11 1 1; every boy is to pay double a girl, every wife double a boy, and every man double a wife: how much pays each?

Answ. A girl 3½d. a boy 7d. a wife 14d. a man 28d.

12. My

13. My brother and I beginning to read through the same book, I beat him the first day 20 pages, the next day 40, the next 60, the next 70, and the last day fairly doubled him, and got through; whereas he had 370 pages to read; I had read the same number every day: I demand the pages in the book?

Answ. 1800. $20 + 40 + 60 + 70 = 190$ and $370 - 190 = 180$ half a day's reading.

C H A P. II.

REDUCTION.

Quest. 1. In L. 48, how many shillings, pence and farthings?

Answ. $48 \times 20 = 960$ s. and $960 \times 12 = 11520$ d. and $11520 \times 4 = 46080$ farthings.

2. In 46080 farthings, how many pounds?

Answ. 48.

$$\begin{array}{r} 4 \overline{)46080} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \overline{)11520 \text{ d.}} \\ \hline \end{array}$$

$$\begin{array}{r} 20 \overline{)960 \text{ s.}} \\ \hline \end{array}$$

$$48 \text{ L.}$$

The first of these is called *Reduction descending*, and the second *ascending*; also these following.

3. In L. 225, how many farthings?

Answ. 216000, descending.

4. In 216000 farthings how many pounds?

Answ. 225, ascending.

5. In

5. In L. 60 19 11 $\frac{1}{4}$, how many farthings?
Answ. 68559.
6. In 68559 farthings, how many pounds?
Answ. L. 60 19 11 $\frac{1}{4}$.
7. In 150 guineas, how many farthings?
Answ. 151200.
8. In 151200 farthings, how many guineas?
Answ. 150.
9. In 500 crowns, how many farthings?
Answ. 120000.
10. In 120000 farthings, how many crowns?
Answ. 500.
11. In 75 moidores, how many farthings?
Answ. 97200.
12. In 97200 farthings, how many moidores, 27s. each?
Answ. 75.
13. In L. 700 Sterling, how many merks Scotch?
Answ. 12600 merks.
14. In 12600 merks Scotch, how many pounds Sterling?
Answ. L. 700.
15. In 500 merks Scotch, how many placks?
Answ. 20000.
16. In 20000 placks, how many merks?
Answ. 500.
17. In L. 175 Sterling, how many pounds Scotch?
Answ. 2100.
18. In L. 2100 Scotch, how many pounds Sterling?
Answ. 175.
19. In L. 200 Sterling, how many Scotch placks?
Answ. 144000.
20. In 144000 placks, how many pounds Sterling?
Answ. L. 200.

It may be observed that 18 merks Scotch make the pound Sterling, 40 placks the merk, and 3 placks are a penny Sterling.

Examples

Examples of Weight.

21. In 36lb. Troy, how many grains?
Answ. 207360.
22. In 207360 grains, how many pounds Troy?
Answ. 36.
23. In 24lb. 10oz. 14dwt. 20gr. how many grains?
Answ. 143396.
24. In 143396 grains, how many pounds?
25. In 900 tons Avoirdupois, how many pounds?
Answ. 2016000.
26. In 2016000lb. how many tons?
27. In 32 stons Dutch, how many drops?
Answ. 131072.
28. In 131072 drops, how many stons?

Examples of Time.

29. How many minutes in an age of 70 years?
Answ. 30792000.
30. How many minutes since the birth of our Saviour?

Examples of Long Measure.

31. How many barley-corns will go round the earth, calling each degree of the Equator $69\frac{1}{2}$. English miles?
32. In 260 guineas, how many pounds Sterling?
Answ. 273.

Rule. Multiplying your guineas by 21 brings them to shillings, and shillings, divided by 20, gives pounds: such questions are called *Mixt* or *Compound Reduction*.

33. In

33. In 246 moidores, how many guineas?
 Answ. 316 guin. 6s.
 34. In 316 guineas and 6s. how many moidores?
 Answ. 246.
 35. In 296 pistoles, at 16s. 6d. how many guineas?
 Answ. 232 + 12s.

Rules. To reduce moidores to pounds, add $\frac{2}{7}$ and $\frac{1}{10}$; to guineas, add $\frac{2}{7}$; and guineas to moidores subtract $\frac{2}{7}$.

36. In L. 245 Sterling, how many ducats, at 3s. 9d. per ducat? Answ. 1306 $\frac{1}{2}$.
 37. In 45 guineas, how many Scotch merks?
 Answ. 810.
 38. In L. 200, how many merks? Answ. 300.
 39. In 1200 merks, how many pounds? Answ. 800?

A merk being $\frac{2}{3}$ of the pound, is reduced to pounds by subtracting $\frac{1}{3}$, and pounds to merks by adding $\frac{1}{2}$.

40. I would exchange L. 420 for half-guineas, nine-shilling pieces, shillings and six-pences, and would have the number of each equal; how many of each sort will I receive?

Answ. 400 of each. Add together the different pieces whereof your exchange is to consist, they make 21s. : so the question is in effect the same, as if it had been asked, In L. 420, how many guineas?

41. A gentleman sends his servant to the bank with L. 7290 in notes; orders him to bring change for the same, in guineas, half-guineas, crowns, half-crowns, shillings and six-pences, and of each an equal number; I demand what number of each the servant must bring? Answ. 3600.

Reduce the given sum to six-pences for a dividend, and take the six-pences in all the different pieces, that is, in a guinea, half a guinea, &c. for a divisor.

42. In

REDUCTION.

31

42. In 925 Portuguese pieces of 36s. how many pounds Sterling? Answ. L. 1665.

You multiply by 2, divide by 10, and subtract the quotient from the dividend.

43. How many guineas in 924 pieces of 36s. each? Answ. 1584.

A short way is, From twice the number of pieces take $\frac{2}{7}$ thereof; for 7 pieces of 18s. make but 6 guineas.

44. How many square-feet in an English acre? Answ. 43560. $22 \times 22 \times 10 \times 9 =$ the answer;

that is, the chain of 22 yards squared, multiplied by 10, the chains in an acre, and by 9, the feet in a yard.

45. The national debt, some time ago, was said to be 145 millions: Suppose this sum all in shillings, and distributed in equal rows, allowing every shilling a square-inch; how many English acres would it cover? Answ. 462 acres, 1 rood, 12 perches, and about 2 yard.

46. Or, supposing the above sum in gold, how many roods of a wall would it build, 20 inches thick? Answ. 4 roods, 8 yards, 8 feet, 672 inches.

Call the guinea 129 grs. and there are 138095238 guineas, so are there 17814285702 grains; then the grains in an inch are 4505, (dropping fractions) so there are 3954336 inches, and these divided by 25920, the inches in a yard of the wall, gives yards, &c.

47. Bought 360 cheefes, weighing $22\frac{1}{2}$ lb. each, how many stones in this cargo? Answ. 506 st. 4 lb.

48. In 500 guineas, and 500 groats,

In 500 pounds, and 500 merks Scots,

In 500 shillings, and 500 crowns;

In Sterling say, what is the total in pounds? Answ. L. 752 15 6 $\frac{1}{2}$.

49. How

49. How oft does the pendulum of a clock vibrate in 24 hours, or in a year's time?

Answ. 86400 times in a day, 31536000 times a year.

50. How oft does the pulse in a human body vibrate during a life of 70 years, supposing it to make 75 strokes in a minute? Answ. 2759400000 times.

51. How many stalks of flax will grow upon an English acre of land, when 9 stalks grow on every square inch? Answer 56453760.

52. We heard once of a carter, who drove his cart 250 miles in 24 hours on a wager; how oft must his wheels have turned round in a minute, supposing them 14 feet, 8 inches in circumference?

Answer 62 $\frac{1}{2}$ times.

53. If a chariot-wheel be 18 feet, 6 inches in circuit, how far would it go in 12 hours, when driven with the rapidity of a millstone, that turns 100 times in a minute? Answer 252 $\frac{1}{2}$ miles.

54. How many frogs were there in Egypt, when they covered all its land, granting there are 90048000 Scotch acres in it, and that 36 frogs can sit upon a square foot? Answer 17751702528000.

55. There is a cubical vessel filled with butter, 3 yards every way; how many stones will this weigh, tron weight, one inch square weighing 8 drops?

Answer 2460 stones, 6 lb.

56. How much weighs the smoke of 100 tons of tobacco, when a pound has 14 ounces of smoke?

Answer 87 tuns, 10 cwt. Sub. $\frac{1}{8}$.

57. From Edinburgh to London is 320 English miles, how long would a person take to measure this road with his foot, supposing him to measure 55 of his feet every minute, and that his foot is 10 inches long? Answer 614 hours, 24 minutes.

58. There are 500 pieces of linen, 33 yards each, to be made into gowns 7 yards each, smocks 3 $\frac{1}{2}$ yards each, petticoats of 3 yards, aprons of 2 yards, and handkerchiefs

handkerchiefs of one yard each; how many ladies will it clothe after this manner?

Answer, 1000 ladies.

59. How many lead drops, 100 in the ounce, may be cast out of a fodder of lead, viz. $19\frac{1}{2}$ cwt.

Answer, 3494400.

60. How many minutes since the creation of the world?

61. How oft has the moon changed since the creation of the world, $29\frac{1}{2}$ days each moon?

Answers to these differ yearly.

62. There are 6 chests of drawers, in every chest 24 drawers, in every drawer 20 divisions, in each division L. 5 13 4; how much money in all?

Answ. L. 16320.

63. There are said to be 1625000 soldiers in Europe at this time; what money will support these a year, at 6d. a-day each man? Answ. L. 14828125.

64. How many hens would it require to pick up a boll of barley in a minute, when each picks up a grain in a second of time, and there are 4365000 grains?

Answ. 72750 hens.

65. A gentleman has about him certain coins, in number and kind as follow, viz. 242 joannes, 170 half ditto, 220 moidores, 170 guineas, 112 half ditto, 60 quarter ditto, 50 crowns, 100 half crowns, 260 shillings, and 500 six-pences; what is the value of his purse? Answer, L. 1777 15.

66. When 20 lacks of rupees make L. 250000 Sterling, the rupee being 2s. 6d. how many are a lack?

Answer, 100000.

67. How many apples, plumbs, and pears,

Will one consume in 20 years,

Who eats 5 apples every day,

Ten pears, and 15 plumbs alway?

Answ. 36500 apples, 73000 pears, 109500 plumbs.

58. In 2667000 dollars, at 4s. 6d. per dollar, (which is said to be the revenue of the king of Portugal) how many pounds Sterling?

Answer L. 600075.

59. Hassan Bey, the noted pirate, who was taken, and impaled at Constantinople in 1773, offered 600000 gold chequins for his life; how much is this in our money? The chequin is worth 8s. 10d. $1\frac{1}{4}$ qr.

Answer L. 265781 3.

70. When Virgil repeated the sixth book of the *Æneid* to Augustus and Octavia, they were so taken with the encomium upon young Marcellus, that they ordered the poet 16 sesteria for every line, and there are 26 lines of it; how much is this in our money? The sesterium is reckoned to have been worth L. 7 16 3.

Answer L. 2031 3.

CHAP. III.

RULE OF THREE.

Quest. 1. If 6 yards of cloth cost 15s. what cost 30 yards?

$$\begin{array}{rcl} Yds. & s. & Yds. \\ 6 & : 15 & : : 30 \\ & 30 & \end{array}$$

$$\underline{6)450}$$

$$75s. = L. 3 \ 15.$$

2. If 15s. buy 6 yards, how many yards will 75s. buy?

15

RULE OF THREE.

35

$$\begin{array}{rclcl} & \text{Yds.} & & \text{s.} & \\ 15 & : & 6 & : : & 75 \\ & & & & 6 \end{array}$$

$$\begin{array}{r} \text{Yds.} \\ 15 \overline{) 450} (30 \end{array}$$

3. If 8 yards linen cost 17s. what cost 40 yards?

$$\begin{array}{rclcl} \text{Yds.} & & \text{s.} & & \text{Yds.} \\ 8 & : & 17 & : : & 40 \\ & & 40 & & \end{array}$$

$$8 \overline{) 680} (85 \text{ s.} = \text{L. } 4 \text{ } 5.$$

4. If 40 yards cost 85s. what cost 8 yards?

$$\begin{array}{rclcl} \text{Yds.} & & \text{s.} & & \text{Yds.} \\ 40 & : & 85 & : : & 8 \\ & & 8 & & \end{array}$$

$$40 \overline{) 680} (17 \text{ s.}$$

5. If 1 yard of cloth cost 3s. 2d. what cost 60 yds.?

$$\begin{array}{rclcl} \text{Yds.} & & \text{s.} & & \text{Yds.} \\ 1 & : & 38 & : : & 60 \\ & & 60 & & \end{array}$$

$$12 \overline{) 2280} \text{ d.}$$

$$20 \overline{) 190} \text{ s.}$$

$$\text{L. } 9 \text{ } 10$$

6. If 60 yds. of cloth cost L. 9 10, what cost 1 yd.?

$$\begin{array}{rclcl} \text{Yds.} & & \text{l.} & \text{s.} & \text{Yds.} \\ 60 & : & 9 & 10 & : : & 1 \\ & & 20 & & \end{array}$$

$$190$$

$$12$$

$$60 \overline{) 2280} (38 \text{ d.} = 3 \text{ s. } 2 \text{ d.}$$

RULE OF THREE.

7. If 4 lb. of cotton cost 3s. 6d. what cost 112 lb.?

$$\begin{array}{r}
 \text{lb.} \quad \text{d.} \quad \text{lb.} \\
 4 : 42 :: 112 \\
 \quad \quad 42 \\
 \hline
 \quad \quad 224 \\
 \quad \quad 448 \\
 \hline
 4)4704
 \end{array}$$

$$1176 \text{ d.} = 98 \text{ s.} = \text{L. } 4 \text{ } 18.$$

8. If 1 yard broad cloth cost 14s. 10d. what cost 70 yards? Answer L. 51 18 10.

$$\begin{array}{r}
 \text{Yd.} \quad \text{d.} \quad \text{Yds.} \\
 1 : 178 :: 70
 \end{array}$$

9. If the rent of an acre of land be L. 5 11 9, what will the rent of 60 acres amount to?

Answer L. 335 5.

$$\begin{array}{r}
 \text{A.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{A.} \\
 1 : 5 \text{ } 11 \text{ } 9 :: 60
 \end{array}$$

10. If 4 yards and 3 qrs. of velvet cost L. 2 7 what cost 24 yards and a half at that rate?

Answer L. 12 2 5 and $\frac{1}{17}$.

$$\begin{array}{r}
 \text{Yds. qrs. l.} \quad \text{s.} \quad \text{Yds. qrs.} \quad \text{Qrs. s.} \quad \text{Qrs.} \\
 4 \text{ } 3 : 2 \text{ } 7 :: 24 \text{ } 2 \quad \text{Or thus: } 19 : 47 :: 98
 \end{array}$$

11. If 1 Cwt. of pepper cost L. 6 14 9, what cost 14 Cwt. 1 qr. 7 lb.? Answer L. 96 8 7 $\frac{1}{4}$ qr.

$$\begin{array}{r}
 \text{Cwt. l. s.} \quad \text{d.} \quad \text{Cwt. qr. lb.} \quad \text{lb.} \quad \text{d.} \quad \text{lb.} \\
 1 : 6 \text{ } 14 \text{ } 9 :: 14 \text{ } 1 \text{ } 7 \quad \text{Or thus: } 112 : 1617 :: 1603
 \end{array}$$

12. If

12. If my wages be 6s. 2d. a-week, how long will I take to gain L. 50? Answer 3 years, 6 weeks.

<i>d.</i>	<i>wk.</i>	<i>d.</i>	<i>Wks.</i>
74	: 1	: 12000	: 162 nearly.

13. What must my wages be per week, to enable me at the year's end to lay up L. 12, to pay for my board and pocket expence L. 10, and lay out upon clothes L. 7 18? Answer 11s. 6d.

<i>Wks.</i>	<i>l.</i>	<i>s.</i>	<i>Wk.</i>
52	: 29	18	: : 1

14. If the interest of L. 100 for a-year be L. 4, what is the interest of L. 34 16 for the same time?

Answer 27s. 10d.

<i>s.</i>	<i>l.</i>	<i>s.</i>
2000	: 4	: : 696

15. What cost 241 good wedders, at L. 11 12 9 the clad score? Answer L. 133 11 1.

16. Bought a parcel of oxen, at L. 14 10 each, intending to sell the beef at 6s. per stone; how much must they weigh a-piece, so as I may have the hides and offals clear gain? Answer 48 stones.

17. A merchant bought 164 yards of cloth for L. 120; for 100 yards of which, being better than the rest, he paid 16s. 6d. a-yard: I demand what the remainder cost per yard? Answer 11s. 8½d.

18. What cost a spindle of black puddings, at a plack per yard? Answer L. 20.

19. A farmer sold a quantity of oats, at 16s. per boll, to be paid on delivery; but then coming to stand in need of money, he proposes to discount every 20th shilling to the buyer, if he will pay it just now; who finding he will thereby save L. 5, agrees to do so: I demand the number of bolls?

Answer 125.

20. If

20. If 24 men, or 30 women, can reap 7 acres of wheat in a day's time, I demand how many acres 16 men and 20 women will reap, all working together?

Answer $9\frac{1}{3}$ acres.

21. A vintner buys L. 140 worth of wine, of two kinds: That of the best kind was 280 gallons, at 4s. the worst kind was 3s. 6d. per gallon: Required the gallons? Answer 480.

22. If the shadow of an English yard measures 4 feet, and at the same time that of a steeple measures 148 feet; what is the height of said steeple?

Answer 111 feet.

23. Whether or not can a large tub, 13 feet in circumference, be taken in at a door 4 feet wide?

Answer No, by 17 inches.

22 : 7 :: 13

24. If 1 lb. of raisins cost $3\frac{1}{4}$ d. what cost 27 fraills, weighing 3 qrs. 18 lb. each? Answer L. 37 5 10 $\frac{1}{2}$.

25. A goldsmith bought a wedge of gold, weighing 14 lb. 3 oz. 8 dwt. for L. 542 15 4; I demand what it stood him per ounce? Answer L. 3 3 4.

26. A grocer bought 8 hogheads of sugar, each weighing 13 cwt. 1 qr. at the rate of L. 2 8 6 per cwt. I demand how much the whole cost him?

Answer L. 257 1

27. If my wages be 1s. 2d. each working day, and my board 3s. 4d. a-week, what shall I have to lay up at the year's end, after laying out L. 5 upon clothes?

Answer L. 4 10 8.

28. A merchant bought 10 packs of cloth, each containing 16 pieces, which, at 7s. per ell Flemish, amounted to L. 1344; I demand how many yards were in each piece? Answer 18 yards.

29. What cost 40 Scotch pints of brandy, at 3d. the gill? Answer L. 8.

30. There are 5 farms to be sold, of equal value. A certain person says, That with half his money he could

could purchase one of them; and if he chuse to borrow L. 1000 he could purchase all the 5; what is the value of one of these farms? Answer L. 133 6 8.

Such a trifle may be proposed to try whether a scholar has his wits about him.

31. A weaver receives for working every yard 4d. and lets a yard drop to the score; how many yards must he work a-day, to enable him to live at the rate of L. 36 10 a-year?

Answer $7\frac{17}{100}$ yards each work-day.

32. A journeyman shoemaker receives 9d. for making each pair of shoes. He spends a-year as follows, viz. L. 7 16 for board; L. 6 16 on clothes, and L. 4 10 on pocket-money; after all, has L. 16 to lay up at the year's end: I demand how many pairs of shoes he makes in a week? Answer 15 pairs.

33. What cost 500 horned brutes,
At crowns the piece for all their clutes?

Answer L. 2000.

34. I got a million rings of brass,
As portion with a bonny lass;
Say, what amount this tocher brings,
At 18d. each gross of rings?

Answer L. 520 16 8.

35. What will be the value of a million of potatoes, that weigh 2 oz. each, and sell at 4d. the stone Dutch? Answer L. 136 4 2.

36. Hired two labourers; the first to have 14d. a-day, and the other 9d. provided they wrought 12 hours a-day; but so it happened, that the last wrought 14 hours a-day, the other but 9, what are the wages of both to be? Answer 10½d. each.

37. How much will it cost to furnish my house with chairs, at 9 guineas per dozen, allowing 16 chairs to each of my 8 rooms? Answer L. 100 16.

38. How many pairs of blankets, 8 yards each, can I purchase for L. 100, at 15d. the yard?

Answer 200 pairs.

39. Sold

39. Sold 40 stons of butter, at 8s. 6d. 60 stons of cheefe, at 4s. 3d. and 30 bolls of barley, at 15s. How many loads of meal must I yet sell, at 21s. to make up L. 62 15, which is the whole of my rent, and public burdens? Answer 10 loads.

40. How long will a plough be in labouring an acre of land, when she makes a furrow 6 inches broad, and goes at the rate of 2 miles an hour?

Answer 8 hours 15 minutes, if English, 9 hours 18 minutes, if Scotch measure.

41. A thief flying every day 40 miles, and intending to embark at a port 600 miles distant; is pursued after 6 days: Queritur, how many miles must his pursuers make good every day, so as to get up with him, by the time he reaches that port?

Answer 100 miles a-day.

42. The Count d'Artois, in 1776, purchased an English horse, at 1700 guineas, what cost every hair of this horse's tail, supposing them 7000 in number?

Answer 5s. 1 $\frac{1}{2}$ d. or suppose 140 hairs more, they cost just 5s. a-hair.

43. Two ships set out to sail round the world, both at one time; and upon the equinoctial, the one sails 144 miles a-day, the other 132: Now, suppose they always keep on the same course, and meet with no impediment, how long will it be ere the foremost overtake the hindermost? (Pardon the incongruity of the expression) Answer 1800 days.

44. In summer 1776, an ox was killed at Glasgow, that weighed 66 stons, and had 14 stons 12 lb. tallow: Now, valuing all at 6d. the pound, and calling the hide and offal 40s. what was the value of this ox? Answer L. 34 6.

45. The annual revenue of King Solomon was 666 talents of gold, what would be the Sterling value of this, at L. 3 17 10 $\frac{1}{2}$ d. per ounce, the talent weighing 114 lb. 15 dwt. Troy?

Answer L. 3549493 16 6 $\frac{3}{4}$.

46. How

46. How many spindles of yarn will it require to make 100 yards of cloth, 18 hundred in the reed?

Answer 59 spindles, 18 cuts.

The proportion is, as the hundreds in the reed : 32, so the spindles to the yards ; or, as 32 : hundreds :: yards : spindles.

47. It is computed that 3000000 pounds tea are annually consumed in England; and with every pound of tea 10 pounds of butter, and 8 of sugar : Now valuing the tea at 5s. the butter at 6d. and sugar at 8d. what is the yearly expence?

Answer L. 2300000.

48. What is the rent of 1800 Scotch acres of land, at one farthing per ell? Answer L. 10800.

49. How many peats that are 4 inches broad, 3 inches thick, and a foot and a half long, may be cast out of 20 square yards of moss, that is 3 yards deep?

Answer 259200.

50. How many yards ribband, an inch broad, will cover an acre of land English; also, what is its value, at 8d. per yard? Answ. 174240 yds. Value L. 5808.

51. How many apples, pears and plumbs, and of each an equal number, can I buy for 20s. when apples are 6, pears 10, and plumbs 30 a halfpenny?

Answer 1600 each.

52. A company of Dissenters having, a mind to collect a sum among them, to build a church, find, that if they pay 5s. a-piece, they will fall L. 20 short of the sum wanted, and if they give 6s. each, they will go L. 30 beyond it; I demand what was the sum wanted? Answer L. 270.

53. A smuggler bought 128 lb. tea, at 7s. per pound; but, meeting with some excise officers, he lost 16 lb. how must he sell the remainder, to lose nothing? Answer 8s.

F

54. A mer-

RULE OF THREE.

54. A merchant buys at Madeira 100 tuns of wine, for L. 438; pays for freight of the same to Britain, L. 250; for loading and unloading, L. 20; for incidental charges, L. 25; for cellar rent, 8s. per tun. He intends to sell his wine, so as to gain L. 200. A vintner offers to buy 36 tuns. Required the charge for these to answer this gain?

Answer L. 350 5 $\frac{1}{3}$.

55. If A can build a house in 12 days, B in 20, and C in 30 days; how long will it serve all the three?

Answer 6 days.

56. If I gain 40s. a-week, how much may I spend, so as to lay up L. 25, together with the expence of 8 weeks, at the year's end? Answer 26s. 4d.

First find the money gained in a year; from which take L. 25; then add 8 to the weeks in a year, and say, 60 : 79 :: 1 : $26\frac{1}{3}$.

57. How many eggs and apples, and of each an equal number, will buy an estate of L. 20000 a-year, at 20 years purchase, when eggs are $1\frac{1}{2}$ d. a-dozen, and apples a penny the score?

Answer 548571428 $\frac{1}{7}$ of each.

Make choice of any number that is a common multiple of 12 and 20; suppose 60: then 60 eggs are worth $7\frac{1}{2}$ d. 60 apples worth 3d. add these, and say, $10\frac{1}{2}$: 60 :: 192000000 (that is the worth of the estate in halfpence) to the answer.

58. A toll-keeper, who paid L. 57 rent for his bar, keeps account of the horses and carriages that pass in a year, being to receive from every horseman a penny, from every cart 2d. and from every coach 4d. Now, at the year's end, he found he had made 19 guineas more than his rent, and observed, that as oft as 3 coaches passed, 10 carts and 25 horsemen passed; I demand how many passed of each?

Ans. 972 coaches, 3240 carts, 8100 horsemen.

Here

Here 3 coaches make 12d. 10 carts 20d. 25 horsemen 25d. in all 57d. your first number, L. 76 19 your second, and your third number is alternately 3, 10 and 25.

59. Bought 288 yards linen and fustian for L. 12 6. A yard of linen was $\frac{1}{4}$ the price of a yard fustian, and the number of yards linen to those of fustian were as 7 to 5. I demand the yards and prices of each?

Answer 168 yards linen at 9d. and 120 yards fustian at 12d. Work thus:

$7 + 5 = 12$; $288 :: 7$: linen; $12 : 288 :: 5$: fustian. Also, to 288 add $\frac{1}{4}$ of the fustian, makes 328; by which divide the money, gives the price of a yard linen.

60. The number of the book, chapter, and verse of a certain text of scripture make up 42, and the number of the verse is quadruple that of the chapter, and the book quadruple that of the verse; what are the words of this text?

Answer *They that observe lying vanities forsake their own mercy.*

$1 + 4 + 16 = 21$; and 42, divided by 21, gives 2, the chapter.

61. Bought 1300 yards of lace, ribband, and incle for L. 26 5. Had 100 yards more of ribband than of lace, and 500 yards more of incle than of ribband. Moreover, a yard ribband cost as much as 12 yards of incle, and a yard lace as much as 20: I demand the yards and prices?

Answ. 200 yards lace, at 15d. 300 yards ribband, at 9d. 800 yards incle at $\frac{3}{4}$ qrs. per yard.

Take 100, the difference of the lace and ribband, 500, the difference of the ribband and incle, and 100 more; for if the incle exceed the ribband 500, it must

must exceed the lace 600; add these, they make 700; and subtract this from 1300, rests 600; divide by 3, gives 200 yards lace, so are the rest known. Multiply the lace by 20, the ribband by 12, which produces yards of incle; add the products with the 800 yards incle, make 8400; by this divide L. 26 5, quotes the value of a yard incle: the rest is easy.

62. A taylor bought 600 yards of serge, shalloon and buckram; which in all cost him L. 135. The quantity of serge was double that of the shalloon, but was 10 times its value; also the quantity of the shalloon was triple that of the buckram, and was 4 times its value: The number of yards of each, and prices are required?

Answer 360 yards serge, at 6s. 8d. 180 yards shalloon, at 16d. 60 yards buckram, at 1s. per yard.

$40 + 4 + 1 = 45 : 135 :: 1 : 3l.$ the value of the buckram.
and $6 + 3 + 1 = 10 : 600 :: 1 : 60$ yards buckram.

63. A farmer sold 30 bolls of barley and 20 loads of meal, for L. 43 2 6; receiving for every 4 bolls barley, as much as for 5 loads of meal: I demand the price of each? Answ. Barley 18s. 9d. meal 15s.

$30 \times 5 + 20 \times 4 : 43 \ 2 \ 6 :: 30 \times 5 :$ value of the barley.

64. A gentleman hires a livery servant for a year, for L. 12, a coat, and a pair of boots. After 8 months they agreed to part, and the servant receives L. 7 and the coat; yet after this, he stays 2 months longer, and receives L. 2 more, with the boots: I demand the value of the coat and boots?

Answer The coat L. 5, boots L. 1.

He wanted only L. 3 of his year's wages, and 2 months were yet to run: this shews his wages to be L. 18 a-year.

See more such questions illustrated in Vulgar and Decimal Fractions.

65. A far-

65. A farmer, who would proportion his seed to his acres, finds, that if he sow 10 bushels on each acre, he shall leave 16 bushels unsowed, and if he sow 12 bushels on an acre, he will leave 4 acres unsowed; Now what quantity shall he sow on an acre, so as to leave neither seed nor land unsown?

Answer $10\frac{1}{2}$ bushels.

Add 16 bushels unsown to 48 wanting; half this, is the number of acres.

66. Bought 5s. worth of apples to distribute among a company of ladies; and giving 10 to each, two were unserved; bought the same quantity again, and then gave each of them 15, and had 30 over: How many did I buy for a crown? Answer 120.

Double 20, the first want; add 30, the overplus at last; divide the sum by 5, gives the number of ladies.

67. Suppose a hare to be 25 of her leaps distant from the hound at starting; that she also makes 6 leaps for every 5 of his, and gains a leap at every doubling, which is every 24 leaps: Now, supposing 4 of the hound's leaps equal to 5 of the hare's, how many leaps must he make before he catch her?

Answer *He will never make any thing of her.*

68. When one pound Troy, of purest gold,
For forty Sterling pounds is sold;
At what rate must we sell a grain,
One shilling on the pound to gain?

Answer $1\frac{1}{4}$ d.

69. I sold 700 pints of brandy,
To Peter, William Tom and Sandy:
Receiv'd from Peter sixty crowns,
From William also twenty pounds,
From Sandy too, (in Spanish cloth)
In value equal to them both;

One-

RULE OF THREE.

One-third part still unpaid, which Thomas
 Stands good for, if he keep his promise:
 How many pints each person bought,
 Together with the price is sought;
 What yards of cloth did Sandy sell me,
 The yard at twelve and six-pence, tell me;
 And further, what does Thomas owe me?
 Take to the pen, and quickly show me.

Answer, Peter bought 100 pints, William $133\frac{1}{3}$,
 Thomas and Sandy each $233\frac{1}{3}$; the price of a pint
 was 3s. you got 56 yards of cloth from Sandy, and
 Thomas owes you L. 35.

70. Gave 5 times 10 of 5 pound notes,
 For wheat and barley, rye and oats;
 The quantities of each were equal,
 The different rates, as in the sequel:
 The wheat at L. 1 5 per boll,
 The barley L. 70 the whole;
 The rye in price exceeds the oats,
 The single boll, by 15 groats,
 Their values both in one compleat,
 By L. 30 exceed the wheat.
 Now by these hints can you attain,
 The bolls and price of every grain?

Answer 60 bolls each; wheat 25s. barley 23s. 4d.
 rye 20s. oats 15s.

From L. 250 subtract L. 70, and L. 30, half the re-
 mainder, is the value of the wheat; by which also
 you find the bolls, the rest will be easy.

THE

INVERSE RULE.

Quest. 1. How many yards of shalloon that is 3 qrs. wide, will serve to line 24 yards of scarlet cloth that is 7 qrs. wide?

$$\begin{array}{rcl}
 \text{Qrs.} & \text{Yds.} & \text{Qrs.} \\
 7 & : 24 & :: 3 \\
 & 7 & \\
 \hline
 & 3)168 &
 \end{array}$$

Ans. 56 yards shalloon.

2. How long will 20 men take to build a house, when 12 such men can do it in 15 days?

Answer 9 days.

$$\begin{array}{rcl}
 \text{Men.} & \text{Days.} & \text{Men.} \\
 12 & : 15 & :: 20 \\
 & 12 & \\
 \hline
 & 20)180 & (9
 \end{array}$$

3. What length of a board, 9 inches broad, will make a foot square? Answer 16 inches.

$$12 : 12 :: 9$$

4. If 6 men can drink a barrel of beer in 20 days, or 10 women can do it in the same time; how long will half the men, and double the women take to drink it? Answer 8 days.

This may be either *direct* or *inverse*.

$$\begin{array}{rcl}
 P. & d. & P. & d. & \text{Men.} & d. & \text{Men.} \\
 5 & : 1 & :: 40 & : 8 & \text{Or,} & 6 & : 20 :: 15
 \end{array}$$

5. There

5. There were 7 men set to work, to cut down a field of grass, which they were able to do in 10 days; but after they had wrought one week, 3 of them gave over: How long will the remaining 4 be in completing the work? Answer 7 days.

6. If the longest end of the beam of a balance be 44 inches, and the shortest 33; how many pounds hung on the shortest will equiponderate 100 on the longest? Answer $133\frac{1}{3}$ lb.

7. A garrison is to be besieged, having in it 840 men, and only 7 days provision; how many men must they dismiss, that the remaining may have 12 days provision? Answer 350 men.

8. Paid for the carriage of 60 stones of cheese, for 60 miles, 25s. how far may I have 100 stones carried for the same money? Answer 36 miles.

9. I have 20 casks of brandy, of 21 pints each; which I am going to retail, and put into bottles of 7 Scotch gills: How many such bottles will I need? Answer 960.

10. I was sent to purchase 160 deals, of 12 feet long, for roofing and flooring a house; but finding none of the size, I would know how many I must buy of those, that are 15 feet long, to answer my commission? Answer 128.

11. I took a field of grass, for the pasture of 40 cows, for 5 months; but as cows happened to be dear, I bought but 24: How long will the pasture serve them? Answer $8\frac{1}{3}$ months.

12. If I have 8 yards, of 4 qr. wide cloth, out of the spindle of yarn, how many yards, of 5 qr. wide, can I have out of the same? Answer $6\frac{2}{3}$ yards.

13. A privateer, of 47 men, had taken a prize of dollars; which being equally divided among them, each had 63, but afterwards 5 of their men dying, a new division was made: I demand how many each would have then? Answer $70\frac{1}{2}$.

C H A P. IV.

DOUBLE RULE OF THREE.

Quest. 1. If L. 100 in 12 months gain L. 5 of interest, what will L. 75 gain in 9 months?

Answ. L. 2 16 3.

L.	M.	L.	M.	L.
100	:	12	:	5
12	:	9	:	75
1200				675

5
 1200)3375(2 16 3

Or thus:

100 : 5 :: 75 : $3\frac{3}{4}$, and 12 : $3\frac{3}{4}$:: 9 : 2 16 3

2. If the carriage of 8 Cwt. 128 miles cost 48s. what cost the carriage of 4 Cwt. 32 miles?

Answer 6s.

Cwt.	Miles.	s.	Cwt.	Miles.
8	:	128	:	48
	:	4	:	32

3. If 4 horses in 7 days eat 16 pecks of corn, what quantity will serve 6 horses 12 weeks?

Answer 18 bolls.

4. Put out L. 730 to interest, and at the end of 90 days received for principal and interest L. 738 2, what was the rate of interest? Answ. $4\frac{1}{2}$ per cent.

5. If a regiment of 548 soldiers in 12 weeks, or 84 days, require for subsistence 60048 lb. of bread; how much bread will it require to sustain 960 soldiers 182 days? Answer 262080 lb.

G

6. If

6. If 12 reapers in 15 days cut 25 acres of corn, how many acres will 70 reapers cut in 24 days?

Answer $433\frac{1}{3}$.

7. If 30 coach-horses consume 84 bushels of oats in 7 days, what quantity will serve 1600 horses the space of a year, at that rate?—And there are at least as many in London. Answer 233600 bushels.

8. Expended in maintenance for my family, when it consisted only of 5 person, 16 guineas per month, or 28 days: What will be my expences now for a year, when my family is increased to 9 persons?

Answer L. 394 4.

9. A house that was 60 feet long, and 24 feet wide, was paved by 5 men in 8 days; again, another house, whose dimensions were in the same proportion, was paved by 12 men in $13\frac{1}{3}$ days: What was the length and breadth of this last house?

Answer 120 feet long, 48 broad.

$60 \times 24 = 1440$ then $5 : 8 : 1440 :: 12 : 13\frac{1}{3} : 5760$ which is the content of the house. Then see Prob. 10. of *Extraction of Roots*.

10. If 4 horses can plough 10 acres of land in 12 days, when the day is 9 hours long, how many acres will 14 horses plough in 6 weeks, when they work $18\frac{1}{4}$ hours a-day?

Answ. 212 acres, 3 roods, 26 falls, 24 ells.

b. d. b. a. b. d. b.

Seven numbers here: $4 : 12 : 9 : 10 :: 14 : 36 : 18\frac{1}{4}$.

Or it may be wrought by three statings of the Single Rule.

11. If the freight of a ship, carrying 500 tons, for a voyage of 8 months, be L. 240, what will be the freight of half a score of vessels carrying each 800 tons for a voyage of 12 months? Answer L. 5760.

12. If 8 cannons, that are 24 pounders, spend 1728 lb. of powder in 20 hours, when they are fired 6 times

6 times an hour ; how many pounds will 14 cannons spend, that are 32 pounders, in 36 hours, when fired 8 times an hour? Answer $9676\frac{4}{5}$ lb.

C. lb. C. lb.

Nine numbers here: $8 : 1728 :: 14 : 3024$ and $24 : 3024 :: 32 : 4032$, and $6 : 4032 :: 8 : 5736$, and $20 : 5736 :: 36 : 9676\frac{4}{5}$. But the terms may be ranked in two columns and abridged as in the following.

13. If 6 lb. of sugar be equal in value to 7 lb. of raisins, 5 lb. of raisins to 2 lb. of almonds, 3 lb. of almonds to 5 lb. of currants, 2 lb. of currants to 18 d. how many pence are the value of 3 lb. of sugar?

Answer 21 d.

lb.		lb.			
6 S.	=	7 R.	Abridged :	1	7
5 R.	=	2 A.		.	.
3 A.	=	5 C.		.	.
2 C.	=	18 d.		.	3
? d.	=	3 S.	and $3 \times 7 =$	21 d.	

Seeing there are 5, 3, and 2 on both sides, they are dropt ; and 6 on the one side, and 18 on the other are divided by 6 : So have we only 7 and 3 to multiply for our answer. But if the question had been, How many pounds of sugar for 21 d. ? then the other column would be the divisor, the terms will stand the same way as before ; only 21 will be last in the left-hand column, and 3 wanting on the right ; and abridging the terms will give 3 the answer.

14. Suppose 5 lb. Avoirdupois, at London, equal to 6 lb. at Amsterdam, 12 lb. at Amsterdam to 13 at Antwerp, 10 at Antwerp to 11 at Lyons, 3 at Lyons to 4 at Genoa, 25 at Genoa to 28 at Millan, 100 at Millan to 63 at Cadiz ; how many pounds at Cadiz are equal to 112 at London?

Answer 150 lb. 10 oz. nearly.

lb.

<i>lb.</i>		<i>lb.</i>	Abridged.	
5 London,	=	6 Amsterdam,	5	1
12 Amsterdam	=	13 Antwerp,	1	13
10 Antwerp	=	11 Lyons,	5	11
3 Lyons	=	4 Genoa,	1	1
25 Genoa	=	28 Millan,	25	28
10 Millan	=	63 Cadiz,	25	21
? Cadiz	=	112 London.	—	28

Then multiplying, divide the product of the right column by that of the left.

15. If 100 yards at London be equal to 160 ells at Leipfic, 100 ells at Leipfic 60 at Geneva, 100 ells at Geneva 55 canes at Rome, 100 Roman canes 303 ells of Holland; how many ells of Holland equal 100 yards at London? Anfw. $133\frac{1}{3}$ nearly.

§ 2. QUESTIONS INVERSE.

Quest. 1. If 12 inches long, 12 broad, and 12 thick make a solid foot, what length of a board, that is 6 inches broad and 4 inches thick, will make the same foot? Answer 72 inches.

$$12 : 12 : 12 :: 6 : 4$$

2. Lent L. 5000, at 5 per cent. for 18 years; at which time recovered only the principal; but the borrower, to make me amends, proposes to let me a farm worth L. 3000 for which he is not to exact any rent, till I be satisfied, and of which I can make 9 per cent. What must the length of this lease be?

Answer 16 years, 8 months.

$$5 : 5000 : 18 :: 9 : 3000$$

3. If it require 6 cuts of yarn of the 10 quarter reel, and sixscore in the cut to be a yard of cloth; how many

many cuts will it require, when the reel is 9 quarters, and fivescore in the cut? Answer 8 cuts.

$$10 : 6 : 6 :: 9 : 5$$

4. Bought 20 yards of cloth, 4 qrs. wide, for L. 12; also 60 yards other cloth for L. 28 10: What will the breadth of this last be, supposing the quality and pennyworth alike? Answer $3\frac{1}{2}$ qrs.

These four questions are wrought by multiplying the two last terms for a divisor; or, if you work by two operations of the Single Rule, both will be inverse; but in those that follow, one will be inverse, the other direct.

5. If L. 100 in one year gain L. 5 interest, what principal will it require to gain 40s. interest in 3 months? Answer L. 160.

L. M. L. L. M.

5 : 12 : 100 :: 2 : 3 Multiply first and last for your divisor.

6. If L. 100 in 12 months gain L. 5 of interest, in what time will L. 160 gain L. 2 at that rate?

Answer, In 3 months.

7. A merchant receives 400 yards of broad cloth, as payment of a bad debt. The cloth is worth no more than 10s. a-yard, but he accepts of it at 18s. 4d. and by that loses L. 100 of his principal, and 4 years interest, at 5 per cent. I demand what was the debt?

Answer L. 333 6 8.

8. How long will it be ere a company of 20 persons drink 20 pints of rum, that carries 7 waters in punch, when every 12 minutes they drink a round of glasses, that hold 8 gills? Answ. 51 hours, 12 min.

9. Twelve millions of wool fleeces are said to be shorn annually in England, how many hands will it employ to shear those in the space of a week, supposing each to shear 100 in a day? Answer 20000.

10. The

10. The annual produce of wheat in England is 35000000 bushels, how many acres will produce this, at 64 bushels an acre? How many reapers will cut it down in 14 days, when 4 can reap an acre per day, and what is the value of this wheat, at 25s. per quarter?

Answ. 546875 acres, 156250 reapers, L. 2734375 the value.

Example of nine Numbers.

11. If 6 reapers, in 8 days, cut down 20 ridges, 200 yards long, and 14 feet broad; how many ridges, 80 yards long, and 16 feet broad, will 10 reapers cut in 12 days? Answer $109\frac{1}{8}$.

Found by two direct and two inverse operations of three numbers.

12. If 60 pioneers cast a trench, 400 yards long, 20 feet wide, and 6 feet deep, in 18 days, when they work 10 hours a-day; how many yards long of a trench will 150 pioneers cast, that is 30 feet wide, and 10 feet deep, in 24 days, working 12 hours a-day? Answer 640 yards.—Here are eleven terms.

C H A P. V.

RULE OF FELLOWSHIP.

Quest. 1. Three merchants, A, B and C, make up a joint stock; A puts in L.60, B, L.80, and C, L.100; they gain L.45: I demand each man's share of the gain? Answ. A gains L.11 5, B, L.15, C, L.18 15.

Add their stocks, and say, 240 : 45 :: 60 and 240 : 45 :: 80, &c.

2. Three

2. Three merchants, A, B and C, are in company; A's stock is L.56, B's L.72, and C's L.96; they gain L.64: I demand each man's share of the gain?

Answ. A's L.16, B's L.20 11 5, C's L.27 8 6 $\frac{1}{4}$.

3. A, B, and C make a joint adventure to Virginia; A puts in hardware to the value of L.600, B cloth valued at L.720, C boots and shoes to the amount of L.840; they jointly pay freight of the vessel, out and home, L.450; and at 6 months end, receive home a cargo of tobacco, value L.4466 neat proceeds: How will this be divided, and what was their gain per cent. per annum?

We are not told here, whether the freight was paid equally, or in proportion to the goods shipped. In either case, the freight each paid, must be added to his stock: In the first case, A's stock would become L.750, B's L.870, C's 990; A's gain L.533 $\frac{1}{3}$, B's L.618 $\frac{2}{3}$, and C's L.704. But in the second case, their stocks would be L.725, L.870, and L.1015; and gains L.515 $\frac{5}{9}$, L.618 $\frac{6}{9}$, and L.721 $\frac{2}{9}$.—They gained 71 $\frac{1}{9}$ per cent. and double that per annum.

4. There are three horses, A, B and C. In the same time that A can eat 5 bushels of oats, B can eat 7, and C 9; in what manner must 52 $\frac{1}{2}$ bushels be divided among them, so as they may begin and end their eating together? Answ. A 12 $\frac{1}{2}$, B 17 $\frac{1}{2}$, C 22 $\frac{1}{2}$.

5. Three merchants, A, B and C, are in company. A puts in L.180, B as much as both A and C; C puts in 200 yards cloth; they gain 15 per cent. A's gain was to C's as 9 to 10. I demand the stock and gain of each?

Answ. A gains L.27, B, L.57, C, L.30; B's stock L.380, C's L.200.

$$100 : 15 :: 180 : \text{A's gain.}$$

$$9 : 10 :: 180 : \text{C's stock.}$$

6. Twenty.

6. Twenty-four lords, 32 lairds, and 40 farmers, are to pay towards a public contribution L. 110; which is to be so proportioned, that a lord is to pay as much as both a laird and a farmer, and a laird is to pay double a farmer: I demand what each will pay?

Anfw. A lord 37s. a laird 25s. a farmer. 12s. 6d.

7. A, B and C put in money together, in all L. 1250. A, L. 50 more than B, and B, L. 150 more than C. A's gain was L. 8 more than B's, and B's L. 24 more than C's: What was the particular stock and gain of each?

Anfw. A's stock L. 500, gain L. 80; B's stock L. 450, gain L. 72; C's stock L. 300, gain L. 48.

The sum of the differences of their stocks is 350 and $1250 - 350 = 900$. This divided by 3, gives the least stock, or C's; the difference of A and B's stocks is L. 50 and of their gains L. 8; so

50 : 8 :: 1250 : 200 the whole gain; and
1250 : 200 :: 300 : 48 C's gain.

8. A, B, C, and D put in money together. A, B and C, L. 447, A, B and D, L. 471, A, C and D, L. 491, B, C and D, L. 505; they gained one-third of their stocks: I demand the stock and gain of each?

Anfw. A, L. 134, gain L. 44; B, L. 145, gain L. 44 6 8; C, L. 168, gain L. 56; D, L. 192, gain L. 64.

$447 + 471 + 491 + 505 = 1917 \div 3 = 639$ the whole stocks, and $639 - 505 = 134$ A's stock, &c.

§ 2. FELLOWSHIP WITH TIME.

1. A, B, and C enter into partnership. A puts in L. 50 for 7 months; B L. 90 for 6 months; C L. 80 for

for 5 months: How much will each receive of the gain, which was L.72.

Answ. A, L.19 10 8 $\frac{1}{4}$, B, L.30 2 9 $\frac{1}{4}$, C, L.22 6 6, a halfpenny lost.

2. Two merchants, A and B, agree to trade together in company for a year; A puts in, January 1st, L.150; March 1st, takes out L.70; June 1st, returns back L.20; October 1st, takes the L.20 out again, the remainder continues to the year's end: B puts in, January 1st, L.60; April 1st, L.60 more; July 1st, L.30 more; September 1st, takes out L.40, the remainder continues to the year's end: Now the gain came out to be L.200, it is required what each person's share will be, in proportion to his stock and time? Answ. A, L.95 18 8 $\frac{1}{4}$, B, L.104 1 3 $\frac{1}{2}$.

Multiply every sum of money by the time it is in company, and add the several products, so will you find for A this total 1180, and for B 1280; these added, are your first number, thus:

2460 : 200 :: 1180 and, 2460 : 200 :: 1280

3. A and B are in company; A puts in L.220, and after 8 months takes out a part; B puts in L.150, and after 6 months L.200 more; at the year's end A's gain was L.60, and B's L.72: I demand what A had taken out at the 8th month's end? Answ. L.35.

Find B's total 3000; then 72 : 3000 :: 60 : 2500, which is A's total;

and 2500 — 1760 (viz. 220 $\times$ 8) = 740 $\div$ 4 = 185 and 220 — 185 = 35 the answer.

4. There is in a cathedral church 20 canons and 30 vicars; these may spend a-year L.2600; but every canon must have, for his part, 5 times as much as every vicar: How much is every man's part?

Answer, The canons L.100, and the vicars L.20 each.—See more of this rule in Decimals.

C H A P. VI.

REDUCTION OF VULGAR FRACTIONS.

Case 1. *To reduce a mixt Number to an improper Fraction.*

Multiply the whole number by the denominator of the fraction; to the product add the numerator; under which place the same denominator: Thus $74\frac{1}{2}$, in an improper fraction, is $\frac{149}{2}$, and $64\frac{1}{8}$ is $\frac{513}{8}$. The same is done with a whole number.

Case 2. *To reduce an improper Fraction to a whole or mixt Number.*

Divide the numerator by the denominator, the quotient is a whole number; if any thing remains, set it after that, with the divisor under it.

Examples. $\frac{12}{3} = 4$. $\frac{13}{2} = 3\frac{1}{2}$ and $\frac{2805}{12} = 233\frac{9}{12}$.

Case 3. *To reduce a Fraction to its lowest terms.*

Divide both parts by any number, that leaves no remainder, and the quotients again in like manner: Thus, $\frac{32}{128} = \frac{2}{8} = \frac{1}{4}$. Here we divided twice by 2, once by 9, and once by 3.

Case 4. *To reduce a compound Fraction to a simple one.*

Multiply all the numerators continually, their product shall be the numerator of the simple fraction; do the same by the denominators: Thus, $\frac{2}{3}$ of $\frac{3}{4}$ becomes $\frac{2 \times 3}{3 \times 4} = \frac{2}{4} = \frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$ will be $\frac{1 \times 3 \times 2}{2 \times 4 \times 3} = \frac{1}{4}$; which may be reduced to lower terms as you please.

Case

Case 5. To reduce Fractions to a common Denominator.

Multiply all the denominators continually for a new denominator, and multiply each numerator by every denominator, except its own, and the products will be the respective numerators. Thus, reduce $\frac{4}{5}$ and $\frac{3}{10}$ to a common denominator; $12 \times 20 = 240$, for the common denominator; $4 \times 20 = 80$, for the first numerator, $2 \times 12 = 24$ for the other; so are the new fractions $\frac{80}{240}$ and $\frac{24}{240}$.

Reduce $\frac{2}{7}$, $\frac{7}{8}$ and $\frac{1}{3}$ to a common denominator.

Here $2 \times 8 \times 13 = 208$, the first numerator, $7 \times 7 \times 13 = 637$ the second, and $12 \times 8 \times 7 = 672$ the third; also $7 \times 8 \times 13$ the denominator; and the three fractions are, $\frac{208}{672}$, $\frac{637}{672}$, $\frac{672}{672}$.

Case 6. To value Fractions in known parts of the Integer.

Multiply the numerator by the next inferior denomination; divide the product by the denominator; the remainder multiply again by the next still lower, &c.

Examples.

What is the value of L. $\frac{5}{8}$ Sterling?

Answ. 12s. 6d.

$$5 \times 20 = 100 \div 8 = 12s. \&c.$$

What is the value of $\frac{3}{4} \frac{2}{8} \frac{0}{0}$ lb. Troy?

Answ. 5oz. 14dwt. 12gr.

$$229 \times 12 = 2748 \div 480 = 5 \text{ oz.}$$

Then multiply the remainder by 20, and divide, you have 14dwt. &c.

§ 2. ADDITION OF VULGAR FRACTIONS.

Reduce the fractions to a common denominator, and add the numerators together, and place their sum over the same denominator.

What

What is the sum of $\frac{1}{2}$, $\frac{4}{8}$ and $\frac{1}{4}$? Answ. $\frac{3}{2}$.

What is the sum of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{7}$? Answ. $\frac{17}{42}$.

Reduce them to a common denominator they are $\frac{21}{42}$, $\frac{8}{42}$ and $\frac{6}{42}$; and these added make $\frac{35}{42}$.

Case 2. If compound fractions are given, they must be reduced to simple ones, and then to a common denominator, and added as before.

What is the sum of $\frac{1}{2}$ of $\frac{1}{3}$, and $\frac{1}{7}$ of $\frac{1}{4}$, and $\frac{1}{11}$ of $\frac{1}{10}$?
Answ. $\frac{17}{2860}$.

When reduced to simple fractions they are $\frac{1}{6}$, $\frac{1}{28}$ and $\frac{1}{110}$; and when brought to a common denominator, $\frac{55}{638}$, $\frac{23}{638}$ and $\frac{6}{638}$; and these added make as above. Again add $\frac{1}{4}$ of a shilling to $\frac{1}{4}$ of a pound.

Note, $\frac{1}{4}$ of a shilling is $\frac{1}{8}$ of $\frac{1}{10}$ of a pound, or $\frac{1}{80}$, and $\frac{1}{4}$ and $\frac{1}{80}$ brought to a common denominator, and added make L. $\frac{21}{80}$.

Case 3. When mixt numbers are given to be added.

Reduce the fractional parts to a common denominator, find their sum; which reduce to a mixt number; add the integral part with the whole numbers given, and to their sum annex the fraction.

What is the sum of $1\frac{1}{2}$, $2\frac{1}{3}$ and $3\frac{1}{4}$? Answ. $6\frac{7}{12}$.

The fractions reduced are $\frac{4}{8}$, $\frac{8}{12}$ and $\frac{3}{4}$; that is, $\frac{6}{8}$, $\frac{8}{12}$; and the sum of the integers is 6.

§ 3. SUBTRACTION OF VULGAR FRACTIONS.

Reduce the fractions every way as in Addition, and take the one numerator from the other, and set the remainder over the denominator.

What is the difference betwixt $\frac{1}{2}$ and $\frac{1}{3}$, or $\frac{2}{3}$ and $\frac{1}{3}$?
Answ. $\frac{1}{3}$.

From $\frac{1}{2}$ take $\frac{1}{3}$ of $\frac{1}{3}$, or $\frac{1}{6}$. Remains $\frac{1}{6}$.

Subtract $\frac{1}{3}$ from 1. Remains $\frac{2}{3}$; for $1 = \frac{3}{3}$.

Case 2.

Case 2. When a fraction is to be subtracted from any other whole number, subtract it from 1, and annex the remainder to the whole number wanting 1.

Take $\frac{6}{7}$ from 13. Rests $12\frac{1}{7}$.

Case 3. If mixt numbers are to be subtracted, and the subtrahend has the smallest fraction, annex the difference of the fractions to that of the integers.

Take $24\frac{1}{3}$ from $36\frac{1}{2}$. Rests $12\frac{1}{6}$.

But if the minuend has the least fraction, take the fraction of the subtrahend from 1, to the remainder add the fraction of the minuend, the sum is the fractional part of the difference, and in subtracting the integers pay 1.

What is the difference betwixt $16\frac{1}{2}$ and $14\frac{2}{3}$?

Answ. $1\frac{4}{3}$.

For $\frac{1}{2}$ from 1, rests $\frac{1}{2}$, and $\frac{2}{3} + \frac{1}{2} = \frac{4}{6}$, and $14 + 1$ from 16, rests 1; so we have $1\frac{4}{3}$. All these might be reduced to improper fractions, and common denominators, and subtracted.

§ 4. MULTIPLICATION OF VULGAR FRACTIONS.

Multiply all the numerators together, and take their product for a new numerator; also the denominators multiplied their product is the denominator. Compound fractions are to be reduced to simple ones, and mixt numbers to improper fractions.

Examples.

Multiply $\frac{7}{8}$ by $\frac{5}{6}$. The product $\frac{35}{48}$.

Multiply $\frac{3}{4}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{1}{6}$; or, $\frac{3}{12}$ by $\frac{1}{6}$; or, $\frac{1}{2}$ by $\frac{1}{6} = \frac{1}{12}$.

Multiply

Multiply $64\frac{1}{2}$ by $28\frac{1}{4}$.

These reduced to improper fractions, are $\frac{129}{2}$ and $\frac{113}{4}$, and the product is $\frac{14589}{8}$, or $1825\frac{1}{2}$.

What is the product of 19s. 11d. 3qrs. multiplied by 19s. 11d. 3qrs.?

These in fractions are $\frac{959}{9}$; so $959 \times 959 = 919681$, and $960 \times 960 = 921600$: Therefore the answer is $\frac{919681}{81}$, or 19s. 11d. $1\frac{1}{9}$ qrs.

§ 5. DIVISION OF VULGAR FRACTIONS.

Prepare the fractions as in last section, then multiply the numerator of the divisor into the denominator of the dividend, the product is the denominator of the quotient; also multiply the denominator of the divisor by the numerator of the dividend, the product is the numerator of it.

Thus divide $\frac{1}{2}$ by $\frac{1}{4}$ $\frac{1}{2})\frac{1}{4}(\frac{2}{1}$ quotient = 2.

Divide $45\frac{1}{2}$ by $20\frac{1}{2}$ $\frac{91}{2})\frac{91}{2}(\frac{4}{1}$, or $2\frac{1}{2}$. Answ.

Divide 24 by $\frac{1}{2}$ $\frac{1}{2})24(\frac{48}{1}$, or 32.

Divide $\frac{1}{2}$ by 15 $\frac{1}{2})\frac{1}{15}(\frac{1}{30}$.

Divide L. 147 among 6 men, so as 5 may have equal shares, the other only $\frac{1}{6}$ share.

$\frac{1}{6})147(\frac{24}{1}$, or L. 28 a share.

Divide L. 375 among 12 men, so as 10 may have equal shares, the 11th $\frac{1}{12}$, and the 12th $\frac{1}{6}$ share.

$\frac{1}{12})375(\frac{31}{1}$, or L. 36.

Divide L. 1260 among 20 men and 20 women, and let every 6 men, have as much as 8 women.

Say, $6:8::20:26\frac{2}{3}$; add 20, is $46\frac{2}{3}$, your divisor.

$\frac{2}{3})1260(\frac{17}{1}$, or L. 27, a woman's share;

Or thus, $8:6::20:15$, and $15+20=35$
 $35)1260(36$, a man's share.

Or, $8+6=14)1260(90 \times 8 = 720 \div 20 = 36$, &c.

§ 6. RULE OF THREE IN VULGAR FRACTIONS.

In the Rule of Three in Fractions, as well as in whole numbers, you multiply the second and third terms, and divide by the first; or, multiply the numerator of the first by the denominators of the other two, and the denominator of the first by the numerators of the other two, which is the same in effect. My own method is to set the denominator of the first term uppermost, and multiply all three together.

Example 1. If $\frac{3}{4}$ of a stone of feathers cost 7s. 3d. what cost $9\frac{1}{2}$ stones? $\frac{3}{4} : 7\frac{3}{4} :: 9\frac{1}{2} : ?$

then $3 \times 240 \times 3 = 2160$ and $4 \times 87 \times 28 = 9744$

that is, $L.\frac{2160}{9744}$ or $L.4$ 10 $2\frac{1}{2}$.

2. If $\frac{1}{2}$ yard velvet cost $L.\frac{2}{3}$, what cost $\frac{3}{4}$ yard?

Answer $L.\frac{2}{3}$.

3. When $\frac{2}{3}$ of $\frac{1}{4}$ of a ship is valued at $L.197$ 11 3, what is the value of the whole ship?

Answer $L.658$ 10 10.

$\frac{2}{3} : 197\frac{11}{12} :: \frac{1}{4} : ?$

4. If $3\frac{1}{2}$ yards of cloth, that is $\frac{5}{8}$ quarters wide, will make a cloke, what length of cloth, that is $\frac{1}{4}$ of a yard broad, will it require?

This is Inverse. $\frac{3}{4} : \frac{5}{8} :: \frac{1}{4} : ?$ These multiplied together make $\frac{15}{16}$; that is, 7 yards.

5. Bought a ring, weighing 3dwt. 20 $\frac{1}{2}$ gr. for which I paid after the rate of coined gold, allowing 5s. for workmanship; what did the ring cost?

Answer, 20s.

Note, 441 guineas a lb. Troy.

6. There is a cistern with 4 cocks, A, B, C and D. A, the largest filling-cock, will fill the cistern in half

an

an hour, and B in 3 quarters; C, the largest emptying cock, will empty the cistern in 40 minutes, and D in an hour: Now, suppose the cistern to be empty, and all the cocks running together, in what time will it be filled? Answer 72 minutes.

A will fill 2, and B $1\frac{1}{3}$ cisterns in an hour, that is $3\frac{1}{3}$; C will empty $1\frac{1}{2}$, and D 1, that is 2.

Now $3\frac{1}{3} - 2 = \frac{5}{3}$ and $\frac{5}{3} : \frac{1}{3} :: \frac{1}{3} : \frac{2}{5}$.

7. From Edinburgh to Glasgow is 44 miles: Now supposing two carriages set out at the same time; one from Edinburgh, travelling 7 miles in 2 hours, the other from Glasgow, travelling 10 miles in 3 hours; in how many hours will they meet, and at what distance from either city?

Answer $6\frac{1}{3}$ hours, $22\frac{1}{3}$ miles from Edinburgh, $21\frac{1}{3}$ from Glasgow.

8. Sound is said to move at the rate of 1142 feet in a second of time: Supposing then, that a great gun be fired at the distance of 40 English miles, how long will it be ere the report be heard?

Answer $3\frac{4}{7}$ minutes.

9. Supposing the velocity of a cannon-bullet to be 1500 feet in a second, at a medium, how much sooner will the bullet reach 6 miles than the sound will?

Answer $6\frac{40}{11}$ seconds.

10. Bought a pair of silver buckles, weighing 30z. 9dwt. 9gr. at 5s. 4d. per ounce; but got discount of $\frac{1}{10}$, the weight for alloy: What did the buckles cost me? Answer 18s. 6d.

11. Received 2748 yards Flander's lace, in payment of a debt of L. 206 2 Flemish; what is the Sterling value of a yard, when L. $\frac{2}{3}$ Flemish is equal to the pound Sterling? Answer 10d. Sterling.

12. A man left his estate to his wife and three sons, as follows: to his eldest son $\frac{1}{3}$, wanting L. 196; to his second $\frac{1}{4}$, wanting L. 148 to his youngest son $\frac{1}{5}$, wanting

98l. and to his wife $\frac{1}{2}$: I demand what his estate was, and the share of each?

Answ. The estate L.1560; the shares, L.584, L.372, L.292, L.312.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12} \text{ and } \frac{13}{12} - 442 = \frac{4}{3}$$

$$\text{and } \frac{13}{12} - \frac{4}{3} = \frac{5}{12} \text{ and } \frac{5}{12} : 442 :: \frac{1}{2}$$

13. Two horses start upon a course of an English mile. The first leaps 15 feet at a bound, and makes 65 leaps in a minute; the other leaps $14\frac{1}{2}$ feet at a bound, and makes 67 leaps in a minute: I demand which horse gains, in what time, and what is the distance?

Answ. The first gains, in $5\frac{1}{3}$ minutes, distance $18\frac{2}{3}$ feet.

14. Bought 16 acres, 2 roods, and 30 falls of oats, at the rate of L.7 13 4 per acre; how many bolls, at 18s. 6d. must I have per acre, so as to gain L.10 on the bargain? Answ. 8 bolls; $14\frac{9}{8879}$ pecks.

15. I bought an ox, a cow and calf,

For 20 guineas and a half.

The cow in value—(let me see)

Was to the ox, as one to three;

The price that for the calf was giv'n,

Was to the cow's, as two to sev'n:

This question then I ask of you,

What cost they each, calf, ox and cow?

Answ. The ox L.15 1 $4\frac{1}{3}$; the cow L.5 0 $5\frac{1}{3}$; the calf L.1 8 $8\frac{2}{3}$.

16. Bought 100 yards fine, and 150 yards coarse linen for L.28 and half a crown. In the sale I gained the same money on the coarse as on the fine, and $44\frac{1}{9}$ per cent. on the whole; moreover, I gained as much on a yard fine, as a yard of the coarse cost me: I demand both buying and selling prices of each?

Answ. The fine, bought at 3s. 9d. sold at 5s. the coarse, bought at 1s. 3d. sold at 2s. 1d.

I

17. Bought

17. Bought a piece of cloth, $\frac{1}{7}$ part of which I took for a suit of clothes, $\frac{1}{10}$ part for a suit to my son, $\frac{1}{7}$ part I sold for L. 2 15, and 12 yards more for L. 3 6; gaining on these sales 10 per cent. I demand how many yards were in the piece, how many are yet on hand, the buying price, and how I must sell a yard of the remaining, so as to make 10 per cent. on the whole, notwithstanding what I have used for clothes?

Ans. 120 yards in all; bought at 5s. $84\frac{1}{2}$ yards yet on hand, which must be sold at 6s. $4\frac{1}{8}\frac{1}{9}$ d. to answer the intended gain.

18. Read $\frac{1}{5}$ of the folios of a large book in a day, and missed 5; next day read $\frac{1}{4}$ of the same, and passed by 20; the third day read $\frac{1}{3}$, and turned over 60; the fourth day read $\frac{1}{2}$ part of them, and passed 80; the last day read $\frac{1}{4}$, and missed 90; and so am got through: Queritur, How many leaves are in this book? Answer 1400.

19. A cuts down 12 acres, B 24 acres, and C a fifth part of a field; after which $\frac{1}{3}$ remains to be cut, which A undertakes to do in 20 days, B in 30, and C in 26 $\frac{1}{3}$; The question is, How many acres there are in the field, and what did each cut in a day?

Ans. 60 acres; A cuts 1 acre, B $\frac{2}{3}$, and C $\frac{1}{4}$ in a day.

20. John, James, and Charles lay out L. 45 each upon linen, holland and cambric: John buys 60 yards of each kind, James 60 yards linen, 74 yards holland, and 50 of cambric; Charles 70 yards linen, 82 yards holland, and 40 of cambric: What cost a yard of each cloth?

Ans. Linen 3s. holland 5s. cambric 7s. a-yard.

Illustration. Seeing James bought an equal quantity of linen, 14 yards more of holland, and 10 less of cambric than John bought, it follows, that 14 yards holland must equal 10 yards of cambric in value; again,

again, comparing Charles's purchase, you'll find 10 yards linen, and 22 of holland must equal 20 of cambric; and, by Proportion, it appears, that 22 of holland are equal to $15\frac{2}{7}$ cambric, thus: $14:10::22:15\frac{2}{7}$; and 20 of cambric wanting $15\frac{2}{7} = 4\frac{2}{7}$, which must be equal to 10 yards linen: Now, $10:4\frac{2}{7}::60:25\frac{2}{7}$ cambric, and $14:10::60:42\frac{6}{7}$ holland, and $60 + 25\frac{2}{7} + 42\frac{6}{7} = 128\frac{2}{7}$, and $128\frac{2}{7}:45::1:\frac{7}{15}$, or 7s. a-yard of the dearest kind, that is, the cambric; the rest is easy.

21. A vintner laid out L. 78 6 upon port at 3s. and claret at 5s. per gallon; and so it was, that the 9th part of his port was equal to the 20th part of his claret in value: What gallons of each did he buy?

Ans. 162 of port, 216 of claret.

$\frac{1}{20}:\frac{1}{9}::\frac{2}{3}:\frac{2}{3}$ so that the claret is $2\frac{1}{9}$, the value of the port, therefore $(2\frac{1}{9})78\frac{6}{10}(24\frac{6}{10})$ the value of the port.

22. Three ships sail west from one place of the equinoctial; A 20 miles, B 32, and C 47 miles a-day: When and where will all three ships come together again?

Ans. In 7200 days, in 120 degrees east longitude.

Find in what time B will overtake A; also, when C will do it: and the least common multiple of these days will be the answer.

C H A P. VII.

REDUCTION OF DECIMAL FRACTIONS.

TO reduce a vulgar fraction to a decimal, the rule is, Divide the numerator by the denominator, annexing

ing ciphers, and continuing the division till nothing remain, or till you think proper to stop.

Examples.

Reduce $\frac{1}{4}$ to a decimal. $4)3.00(.75$ Answ. Finite.
 Reduce $\frac{1}{8}$ to a decimal. $8)1.000(.125$ Also finite.
 Reduce $\frac{1}{16}$ to a decimal. $16)1.0000(.0625$

Your decimal must consist of as many places as you annex ciphers, which may occasion you sometimes to prefix one or more ciphers to your decimal.

Reduce $\frac{1}{3}$ to a dec. $3)1.000(.333$ A pure repeater.
 Reduce $\frac{1}{6}$ to a dec. $6)1.0000(.1666$ A mixt repeater.
 Reduce $\frac{1}{7}$ to a dec. $7)1.000000(.142857$, A pure circ.
 Reduce $\frac{1}{14}$ to a dec. $14)3.000, \&c.(.2,142857$, A mixt circulate,

Case 2. *To reduce the parts of Coin, Weights, Measures, &c. to Decimals.*

Convert the given part to a vulgar fraction, which reduce as above.

Examples.

Reduce 8s. to a decimal; that is, $\frac{8}{20}$. Therefore,
 $20)8.0(.4$ A finite decimal.
 Reduce 6d. to a decimal, viz. $\frac{6}{240}$, or $\frac{1}{40}$.
 $40)1.000(.025$ Finite.
 Reduce 19s. $11\frac{1}{4}$ d. to the decimal of a pound.

You may bring all this to farthings, and divide by 960; or more expeditiously, divide the farthings by 4, thus: $4)3.00(.75$ Before this quotient set your pence, and divide by 12, thus: $12)11.75(.9791666$ Before this again set your shillings, and divide by 20)19.9791666(.99895833 This is a mixt repeater.

Reduce

Reduce 5 oz. to the decimal of a lb. Troy.

$$12)5.00, \&c. (.4166$$

Reduce 1 week to the decimal of a year.

$$52)1.00, \&c. (.01,923076,9 \quad \text{This is a mixt circulate.}$$

Case 3. To reduce a Decimal to its primitive Vulgar Fraction.

If the decimal is finite, set 1 under the decimal point, with a cipher under every part of the decimal, and this is the vulgar fraction; which may be reduced to lower terms, thus: .75 may be made a vulgar fraction, thus: $\frac{75}{100}$, and brought to its lowest terms, is $\frac{3}{4}$.

If the decimal be a pure repeater or circulate, set 9 for the denominator of the repeater, and 9 for every figure in the circle.

Thus a repeating .3 is $\frac{3}{9}$, or $\frac{1}{3}$, and .36, a circulate, is $\frac{36}{99}$, or $\frac{4}{11}$.

If the fraction be mixt, whether repeating or circulating from the whole fraction, subtract the finite part, the remainder is the numerator of the vulgar fraction; and the denominator is 9 for each figure of the circle, or 9 for a single repeater, with a cipher for every place in the finite part.

Example. Reduce .9,27, to a vulgar fraction.

$$.9,27,$$

$$\underline{9}$$

$$918$$

$$\underline{990}$$

or $\frac{918}{990}$, the vulgar fraction.

So will .16 be reduced to $\frac{16}{90}$ or $\frac{8}{45}$.

Case 4.

Case 4. *To reduce a Decimal to the known parts of Coin, Weights or Measures.*

Multiply the decimal by the number of parts contained in an integer; point off the same number of decimals, the figures to the left are the parts, and the decimal is now of a lower denomination, and may be multiplied again for lower parts.

Examples.

What is the value of L. .678 Sterling?

What is the value of L. .725 Sterling?

What is the value of L. .0125 Sterling?—See the work of these:

$.678 = 13s. 6\frac{1}{2}d.$	$.725 = 14s. 6d.$	$.0125 = 3d.$
29	20	20
13.560	14.500	0.2500
12	12	12
6.720	6.000	3.0000
4		
2.880		

Examples.

Reduce .125 cwt. to known parts 14lb.

Reduce .735 lb. Troy to known parts. 8oz. 14dwt.

Reduce .125 years to known parts. 6wks. 3da. 12h.

$.125 \text{ cwt.}$	$.735 \text{ lb.}$	$.125 \text{ years.}$
4	12	52
.500	8.700	6.500
28	20	7
14.000	14.000	3.500
		24
		12.000

English money may be found by inspection, thus: Double the figure next the point, gives shillings; if the next figure be 5 or more, add 1 to the shillings; then the second figure, if under, or its excess above 5, with the third, make farthings; but if 25 or above, reject 1.

Case 5. To reduce Decimals to different forms.

Circulates of the same number of places are said to be similar, and when they begin and end at the same distance from the point, they are said to be conterminous. Circles may be made similar and conterminous, thus: Extend every circle to as many places as is the least common multiple of all the circles, and so they will be similar; then set off as many figures for a finite part as are in the longest finite, and extend the circles as before: So will they be both similar and conterminous.—See Examples:

<i>Disimilar Circles.</i>	<i>Made similar.</i>	<i>Made both similar and conterminous.</i>
.475	.475475	.47,547547
.3,24	.3,242424	.32,424242
.59	.5959595	.59,595959

§ 2. ADDITION OF DECIMALS.

In finite or approximate decimals work as in Integers.

<i>These finite.</i>		<i>Approximates.</i>	
64.15	.75	.0312 +	.0313 —
28.175	.895	.0187 +	.0188 —
30.1	.5	.0406 +	.0406 +
60.2375	.225	.0281 +	.0281 +
12.0125	.95	.0437 +	.0438 —
194.675.	3.32	.162	.162 certain.

Case 2.

Case 2. In adding repeaters, carry by 9 in the right-hand column; if finites are with them, give the repeaters a place more.

.666	246.125	1.25
.25	16.3333	2.75
.333	24.7777	4.333
.555	156.2222	1.666
.444	12.1111	2.166
2.25 finite.	455.5694 mixt.	12.166 repeaters.

Case 3. When circulates are to be added, make them similar and conterminous; if finites are with them, make the finite parts of the circles equal to the longest finite; if repeaters are with them, make them conterminous with the circles; and in adding, take the carriage from the left column of the circles.

.125	12.42,542542
.142,857142	12.27,272727
.545,454545	36.25
.181,818181	22.37,437437
.666,666666	23.12,612612
1.661,796536	106.44,865319

§ 3. SUBTRACTION OF DECIMALS.

Place the numbers as in Addition; in finites or approximates, work as in Integers.

From	.125	.5	.23764 +	.235 —
Take	.0725	.245	.14286 +	.124 +
Rem.	.0525	.255	.0947	.11

Case 2.

DECIMAL FRACTIONS. 73

Case 2. In repeaters borrow 9 on the right. Observe the same directions, as in Addition, in placing the numbers.

24.273	42.125	36.2222	2.4533
16.666	14.4444	13.775	1.2633
7.506	27.6805	22.4472	1.19

Case 3. In circulates pay 1 on the right hand, if you have to borrow on the left.

42.428571	12.27,272727	4.66	1.27
34.857142	10.37,437437	.81	.77
7.571428	1.89,835289	3.84	.49

§ 4. MULTIPLICATION OF DECIMALS.

Give as many decimal places to the product as are in both factors ; and if you have not so many, prefix ciphers.

.125	68.5	24.117 +	24.12 —
.25	2.5	.3529 +	.353 —
Prod. .03125	171.25	8.5108893	8.51436

Case 2. When a finite multiplier multiplies a repeater, carry by 9 on the right hand ; and before you add, make the repeaters conterminous.

246.3	42.16	36.126
2.5	2.86	.086
12316	25300	216760
49266	337333	2890133
	843333	
615.83	120.5966	3.106893

K

Case 3.

Case 3. When a finite multiplier multiplies a circulate, bring the carriage from the left hand; and before you add, make the circles conterminous.

$\begin{array}{r} .571428 \\ 48 \overline{) 4571428} \\ 22857142 \end{array}$	$\begin{array}{r} 148.27 \\ .25 \\ 74136 \\ 296545 \end{array}$
27.428571 a pure circle.	37.0681 a mixt circle.

Case 4. When a Repeater multiplies a finite Multiplicand.

In this and all cases, when the multiplier is infinite, reduce it to a vulgar fraction; then multiply by the numerator of that fraction, (as in Cases 1, 2 or 3) according as the multiplicand is finite, repeating or circulating, and divide the product by the denominator, which will always be some number of 9's.

$\begin{array}{r} 72.25 \text{ by } 2.6 \\ 2.6 \overline{) 9)43350} \\ 48.16 \\ 14450 \\ 192.66 \end{array}$	Or thus: 72.25 $\begin{array}{r} 24 \\ 28900 \\ 14450 \\ 9)1734.00 \\ 192.66 \end{array}$	$\begin{array}{r} 2.6 \\ 2 \\ 24 \end{array}$
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Case 5. When both Factors repeat.

$\begin{array}{r} 243.7 \text{ by } 2.8 \\ 26 \quad 2 \\ 14746 \quad 26 \\ 49155 \\ 9)6390.2 \\ 710.024 + \end{array}$	$\begin{array}{r} 37.6 \text{ by } 9.3 \\ 84 \quad 9 \\ 1506 \quad 84 \\ 30133 \\ 9)3164.0 \\ 351.55 \end{array}$
--	---

Examples

Case 6. When a Repeater multiplies a Circulate.

56.27 by 3.6	98.45 by 8.6
<u>33</u> <u>3</u>	<u>78</u> <u>8</u>
16881	78763
168818	689181
9)1857.00	9)7679.45
<u>206.33</u>	<u>853.27</u>

Case 7. When a Circulate multiplies a Finite.

567.75	345.25 by 3.72
<u>.27</u>	<u>369</u> <u>3</u>
397425	310725
113550	207150
<u>153.2925</u>	<u>103575</u>
1 5473	1273.9725
<u>1</u>	12.8582
154.840	<u>1 1</u>
	1286.8408

Case 8. When a Circulate multiplies a Repeater.

234.6	96.3
<u>.18</u>	<u>.36</u>
18773	5780
23466	28900
<u>42.240</u>	<u>34.680</u>
426	340
<u>42.666</u>	<u>1</u>
	350303 +
	Examples

Case 9. *When both Factors circulate.*

$\begin{array}{r} 321.135 \text{ by } 4.45 \\ \underline{441} \\ 99)1416.20594 \\ \underline{} \\ 1430.51105 \end{array}$	$\begin{array}{r} 4 \\ \underline{4} \\ 441 \end{array}$	$\begin{array}{r} 963.405 \text{ by } 7.18 \\ \underline{711} \\ 99)6849.81243 \\ \underline{} \\ 6919.00245 \end{array}$	$\begin{array}{r} 7 \\ \underline{7} \\ 711 \end{array}$
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§ 5. DIVISION OF DECIMALS.

Work as in Integers, and give as many decimal places to the quotient as the dividend has more than the divisor.

Examples.

$$\begin{array}{l} 6.15)55.35(9 \quad 24.5)147.0(6 \quad 64.5)32.25(.5 \\ 18.5)115.625(6.25 \quad 625)9.125(.0146 \\ 24.2468 +)342.357864(14.119 \text{ \&c.} \end{array}$$

Case 2. If the dividend repeat, work as before, and and in continuing the division, annex the repeating figure.

$$\begin{array}{r} 26).444(.017094,0 \quad 65.)378.55(58.23916,2 \\ \underline{26} \\ 184 \\ \underline{182} \\ 24, \text{ \&c.} \end{array}$$

Case 3. If the dividend circulate, continue the division by annexing the figures of that circle as they come in course.

$$23.5).307692(.01309 \quad 567.25)234353.675(413.14001$$

In

In all cases, when the divisor is interminate, reduce it to a vulgar fraction; then multiply the dividend by the denominator of that fraction; and divide the product by its numerator. Follow examples of every case. And first, the divisor a repeater, the dividend finite.

$$\begin{array}{r} 24.3)9654.25 \\ \underline{24 \ 965425} \\ 219)86888.25(396.75 \end{array} \quad \begin{array}{r} 35.6)24678.25 \\ \underline{35 \ 2467825} \\ 321)222104.35(691.9135 \end{array}$$

Case 5. When both Factors repeat.

$$\begin{array}{r} 42.7)123456.8 \\ \underline{42 \ 123456} \\ 385)111111.2(2886.00519 \end{array} \quad \begin{array}{r} 20.3)100.16 \\ \underline{20 \ 1001} \\ 183)901.5(4.9262 + \end{array}$$

Case 6. When a Repeater divides a Circulate.

Divide L.78.27 among 4 men and 2 boys, the men to have equal shares, but the boys $\frac{1}{2}$ share each.

$$\begin{array}{r} 4.6)78.272727 \\ \underline{4 \quad \quad \quad 9} \end{array}$$

$$42)704.454545(16.772,72, \&c. \quad L.16 \ 15 \ 3\frac{1}{4}$$

Case 7. When a Circulate divides a Finite Dividend.

$$\begin{array}{r} 6789.000000 \\ \underline{6789} \\ .714285)6788.993211(9504.6 \end{array} \quad \begin{array}{r} 246.00 \\ \underline{246} \\ .27)243.54(902. \end{array}$$

Case 8. When a Circulate divides a Repeater.

$$\begin{array}{r} 24.72)64.66 \\ \underline{24 \quad 64} \\ 2448)64.02(2.615196 \end{array} \quad \begin{array}{r} 78.63)248.66 \\ \underline{78 \quad 248} \\ 7785)246.18(3.16223 + \end{array}$$

Case 9.

Case 9. *When both Factors circulate.*

24.45)624.72

24 624

624.162)9874.324

624 9874

2421)618.48(25.5464

623538)9864.450(15.82012

C H A P. VIII.

RULE OF THREE IN DECIMALS.

Quest. 1. If 17 yards of cloth cost L.19 2 6, what cost 35 yards at that rate? Answer L.39 3 6.

Yds. L Yds. L
17 : 19.125 :: 35 : 39.375

2. If the clothing of a regiment of 740 soldiers comes to L.3000, how much is that each man?

Answer L.4 1 0 $\frac{1}{4}$.

74 : 3000 : 1 : 4.054,

3. What cost 6174 table-knives, at 4s. 4d. per dozen? Answer L.111 9 6.

12 : .216 :: 6174 : 111.475.

4. If 1 ounce cost 8d. what cost 6 $\frac{1}{2}$ stoncs Dutch?

Answer L.55 9 4.

oz. l oz. l
1 : .03 :: 1664 : 55.46

Stone. l Stones. l

Or thus: .00390625 : .03 :: 6.5 : 55.46

5. A Draper

5. A draper bought 4 bales of cloth, at L.32 8 per piece, and there were 12 pieces in a bale, and 54 yards in a piece; what was the amount of the whole, and what the price of a yard?

Answer L.1555 4 the amount, at 12s. a-yard.

$$1 : 32.4 :: 48 \text{ and } 2592 : 1555.2 :: 1 : .6$$

6. Bought a quantity of Scotch cheese, at 5s. 8d. per stone, being allowed a pound to the stone; what will the whole cost me, viz. 200 stone neat?

Answer L.53 6 8.

$$1 : .283 :: 200 : 56.6 - 3.3 \text{ the 17th part.}$$

7. If 5 ounces of silk cost 7s. what cost 18 lb.?

Answer L.20 3 $2\frac{1}{5}$.

$$.3125 : .35 :: 18 : 20.16$$

8. An ingot of silver weighs 5lb. 10oz. 14dwt. what is it worth, at 5s. 2d. per ounce?

Answer. L.18 5 3 $\frac{3}{4}$.

$$.083 : .2583 :: 5.8916 : 18.264$$

9. A man bought a piece of cloth, which cost L.33, at 15s. per yard; how many yards did it contain? Answer 44 yards.

$$.75 : 1 :: 33 : 44$$

10. Bought 7 yards scarlet for L.4 12, for a suit of clothes; but being offered the whole piece 6d. a-yard cheaper, if I would take it all; I am resolved to do so, what will it amount to, being 40 yards?

Answer L.25 5 $8\frac{1}{2}$.

$$7 : 4.6 :: 40 : 26.2857 \text{ From which take L.1.}$$

11. If I have an estate of L.456 15 per annum, at what rate may I live per day, so as to lay up L.150 at the year's end? Answer 16s. $9\frac{1}{2}$ d. per day.

$$456.75 - 150 = 306.75$$

$$\text{Then } 1 : 306.75 :: .02739 + : .84$$

12. How

12. How many mole-skins, 6 dozen whereof make a square yard, will line a cloke, that has in it 4 yards of cloth, 54 inches broad? Answer 432.

Inverse. $1.5 : 4 :: 1 : 6$ and $72 \times 6 = 432$.

13. How many planks, that are 14 feet long, and $1\frac{1}{2}$ foot broad, will cover a floor, that is 60 feet long, and 20 feet wide? Answer $57\frac{1}{2}$.

$20 : 60 :: 1.5$ Inverse. The answer is 800 feet; which divide by the length of a plank.

14. At the rate of 5 per cent. per annum, what is the interest of L.60 for 40 days? Answer 6s. $6\frac{1}{4}$ d.

L. $\mathcal{R}$. L. L. $\mathcal{R}$. L.

100 : 1 : 5 :: 60 : .1095 : .3285

15. At what rate per cent. per annum will L.500 gain 5s. per week? Answer $2\frac{1}{2}$ per cent.

16. If 14 bushels of oats serve 16 horses 4 days, how many horses will eat 441 bolls in a week?

Answer 4608 horses.

17. Suppose when wine is at L.30 per tun, that 10s. worth will satisfy 6 men; how many men will be sufficed with L.5 worth, when wine is at L.25 per tun? Answer 72 men.

18. Bought a quantity of lawn for L.46, at 5s. 9d. per ell Flemish; I demand how many yards English there are, and the price of a yard?

Answer 120 yards, at 7s. 8d. per yard.

19. A shepherd, who had 6 wedders to dispose of, desires a storemaster to sell them for him, who, putting them among 48 of his own, delivers them to a drover, to sell for him to the best advantage, allowing him, for his pains, 2d. on each of them, and $2\frac{1}{2}$ sheep by and attour; the drover sells them to a butcher, at 10s. each 50 of them, and gives the other 4
to

to the bargain : Now I desire to know what the shepherd should receive for his 6?

Answer L.2 11 11 $\frac{3}{4}$.

$50 \times 10 = 500$ L.25 and $54 : 25 :: 2.5 : 1.1574$
add 54 two-pences is 1.6074 and $25 - 1.6074 =$
23.3925 then say $54 : 23.3925 :: 6 : 2.5991$

20. There are 67 acres of wheat to be reaped by 30 persons. The 10 best hands can reap 25 acres in 8 days, the next 10 can reap 11 acres in 4 days, the last 10 can reap 5 acres in 2 days; how long will the 67 acres serve the 30? Answer 8 days.

21. B lends to A upon bond, bearing interest from the date, at 5 per cent. L.560. At the end of 7 years A breaks; B is therefore obliged to compound for his debt, and receives of principal and interest 17s. in the pound: It is required how much B will recover, and how much he is a loser?

Answer, Recovers L.642 12, loses L.113 8.

22. Bought a cargo of sheep-skins, and nolt-hides, for which paid L.100 Sterling, at the rate of 9s. for every hide, and 1s. for every skin, as the seller asserted (for I did not number them): He withal told me, I had 100 more skins than hides; this was true; but though I sold the hides at 10s. 6d. and the skins at 14d. each, yet I received but 100 guineas for the whole; I am therefore certain I paid for more than I received: Can you tell how many the seller has miscounted me? Answ. 20 of each.

For 100 skins = L.5 and $100 - 5 = 95$. Now on this L.5 you gain 16s. 8d. and on the rest thus: $10 : 11.66 :: 95 : 110.83$ Now $L.105 - L.5 = 100$ = L.99 3 4 and $110.83 - 99.16 = 11.6$ what you want of the profit you should have had, and 10s. 6d. + 14d. = L.583 and $11.6 \div .583 = 20$ the answer.

L

23. Bought

23. Bought 50 packs of Galloway wool, 11 stones each, 26lb. in the stone, at 6s. 8d. paid 8s. per pack carriage to Ayr, sold it at 7s. 6d. per stone of 24 lb. What was my clear gain? Answ. L. 20 2 1.

24. A bankrupt offers his creditors 15s. a-pound, which they refuse, telling him, they knew his subject very well, and that he would have L. 100 left after paying them 15s. a-pound; they therefore insist for 16s. But, says he, I know my subject will fall L. 100 short of paying you so much: I demand what was his debt and subject?

Answer, His debt L. 4000, his subject L. 3100.

25. A general of an army, after a fore battle, finds, on reviewing his men, that $\frac{1}{3}$ of them had been slain, $\frac{1}{6}$ taken, and that $\frac{1}{6}$ of them have deserted, so that now he has only 15600 left him; how many were in his army when complete? Answer 24000.

26. A grazier buys, at May-day, 40 oxen, at L. 7 12 each; he pays for grass L. 75; he sells the 20 best, at Hallow-day, for L. 11 5 each, and the other 20 at L. 9 15; incidental charges amounted to L. 5 14 6: What was his clear gain? Answer L. 35 5 6.

27. A man bequeaths his subject equally to his 4 sons; but living 7 years after this, and one of his sons being dead, and his money increased by the interest, at 5 per cent. he leaves it equally to the three survivors, and each had L. 240 more than at first; I demand what was the subject at first? Answ. L. 1200.

Add $\frac{1}{3}$ of $\frac{1}{4}$ to $\frac{7}{10}$, which, in a decimal, is .2, and say, .2 : 240 : 1

28. George Goldie and Samuel Silver have to pay a certain sum equally betwixt them, which they do as follows: First, $\frac{1}{3}$ of it, and Goldie pays a guinea as oft as Silver pays a shilling; again, they pay $\frac{1}{2}$ the sum, and Goldie pays a guinea for Silver's crown; and at the last payment Silver has to pay L. 524, part of it to the

the creditor, and a part to Goldie: I demand the sum thus paid? Answ. L.858.

It will be as commodious to work this by Vulgar Fractions. Silver pays at first $\frac{1}{4}$ of $\frac{1}{3}$, that is, $\frac{1}{12}$; and again, $\frac{1}{8}$ of $\frac{1}{3}$, or $\frac{1}{24}$, that is in all, $\frac{3}{24}$; and this wants $\frac{1}{24}$ of his $\frac{1}{3}$. Goldie pays at first $\frac{1}{6}$; and again, $\frac{1}{12}$; in all, $\frac{2}{12}$, that is, $\frac{1}{6}$ more than his $\frac{1}{6}$; which, added to Silver's deficiency, makes $\frac{1}{24}$; and then say, $\frac{1}{24} : 1 :: \frac{1}{6} : 858$.

29. If A and B can do a piece of work in 15 days, A and C in 20 days, and B and C could do the same in 30 days; in what time would any one of them do it?

Answ. A in 24, B in 40, and C in 120 days.

A and B do the 15th part in a day, A and C the 20th, B and C the 30; in all .15. Half this .075, is what all three will do in a day; subtract .033, what B and C do, and there remains .0416, A's work a-day, and .0416 : 1 :: 1 : 24 &c.

30. A herring, a hen, and a ham

Were set on the table before us;

A bottle, a bowl and a dram,

For shillings a-head, half a score o's.

The fish was a fifth of the fowl,

And both were a fifth of the ham;

The bottle a fifth of the whole,

And so were the bowl and the dram.

We paid for our hunger and thirst

In equal proportion, to wit,

As article last to the first,

In order as formerly wrote:

Exhibit a sensible state

Of this very nice bill of fare,

Perhaps, if alive at this date,

It ev'n might perplex Mr. MAIR.

Answ. The herring 2d. the hen 10d. the ham 5s. the bottle 2s. the bowl 1s. 9d. dram 3d.

It

It is easy to see the eating part was 6s. and the drink 4s. and since the fish and fowl were $\frac{1}{3}$ of the ham, they must be $\frac{2}{3}$ of the whole eating; and as the fish was $\frac{1}{3}$ of the fowl, it would be the $\frac{1}{6}$ of a shilling, &c.

§ 2. PARTNERSHIP.

Quest. 1. A, B and C purchase and embark at the Canaries for London, 216 hhds. of wine; whereof A's part is 96, B's 72, and C's 48 hhds. but meeting with a storm, they were obliged to throw over board 90 hhds. How many lost each?

Answer, A lost 40, B 30, and C 20 hhds.

2. A and B are in company; A puts in L.115 more than B; A's money was in company $9\frac{1}{2}$ months; and B's $13\frac{1}{3}$; they divided the gain equally: I demand the stock of each?

Answer, A, L.400, and B, L.285.

$$3.83 : 115 :: 13.3 : 400 \quad 3.83 : 115 :: 9.5 : 285$$

3. Two merchants put in L.990 together, and gained 20 per cent. A's stock was to B's gain as 17 to $2\frac{2}{3}$, B's stock was to A's gain as 13 to $3\frac{1}{3}$; Required the stock and gain of each?

Answer, A's stock L.561, gain L.112 4; B's stock L.429, gain L.85 16.

$$17 + 13 = 30 : 990 :: 17 : A's \text{ stock};$$

$$\text{or, } 6 : 198 :: 3.4 : \text{his gain.}$$

4. A puts in $\frac{2}{4}$ of the stock, and takes $\frac{1}{3}$ of the gain; B puts in L.600, and takes $\frac{1}{4}$ of the gain; C put in for 8 months, and takes $\frac{1}{5}$ of the gain; D puts in L.104 for 10 months, and takes L.52 of gain: I demand stock, gain, and time of each?

Answer A's stock L.276.6, time 5.8 months, gain L.80; B's stock L.600, time 2 months gain L.60; C's stock L.120, time 8 months, gain L.48.

5. A, B, C, and D ship off goods for Virginia; A to the value of L.560, B, L.680, C, L.960, and D, L.800; they have in return 100 hhds. tobacco, at L.30 per hoghead; 500 barrels of tar, at 25s. per barrel, and a remittance of 5000 dollars, at 4s. A takes 20 hhds. of the tobacco, and 100 barrels of tar; B 24 hhds. and 200 barrels; C and D each 28 hhds. and 100 barrels: How must the dollars be shared among them, to even their accounts?

Answer, A must have 691.6, B 391.6, C 2575, and D 1341.6 dollars.

CHAPTER IX.

PRACTICE.

What cost 600 yards, at 3d. 4d. or 6d. per yard?

at 3d.

80)600

L. 7 10

at 4d.

60)600

L. 10

at 6d.

40)600

L. 15

What cost 572 yards, at 5d.?

Thus.

60)572

4) 9 10 8
2 7 8

L. 11 18 4

Or thus.

80 }
120 } 572

7 3
4 15 4

L. 11 18 4

Or thus.

40)572

6)14 6
2 7 8 Sub.

L. 11 18 4

What ✓

What cost 240 yards, at 7d.?

Thus.	Or thus.	Or thus.
$\begin{array}{r} 80 \overline{) 220} \\ 2 \ 13 \\ \underline{3 \ 13 \ 4} \\ \text{L. } 8 \ 8 \ 4 \end{array}$	$\begin{array}{r} 40 \overline{) 220} \\ 5 \ 10 \\ \underline{18 \ 4} \\ \text{L. } 5 \ 8 \ 4 \end{array}$	$\begin{array}{r} 60 \overline{) 220} \\ 3 \ 13 \ 4 \\ \underline{2 \ 1 \ 16 \ 8} \\ \text{L. } 3 \ 8 \ 4 \end{array}$

What cost 250 yards, at 10d.?

Thus.	Or thus.	Or thus.
$\begin{array}{r} 30 \overline{) 250} \\ 8 \ 6 \ 8 \\ \underline{2 \ 1 \ 8} \\ \text{L. } 10 \ 8 \ 4 \end{array}$	$\begin{array}{r} 40 \overline{) 250} \\ 6 \ 5 \\ \underline{4 \ 3 \ 4} \\ \text{L. } 10 \ 8 \ 4 \end{array}$	$\begin{array}{r} 20 \overline{) 250} \\ 12 \ 10 \\ \underline{2 \ 1 \ 8} \text{ Sub.} \\ \text{L. } 10 \ 8 \ 4 \end{array}$

What cost 537 yards, at 11d.?

$\begin{array}{r} 60 \overline{) 537} \\ 8 \ 19 \\ \underline{8 \ 19} \\ 6 \ 14 \ 3 \\ \text{L. } 24 \ 12 \ 3 \end{array}$	$\begin{array}{r} 40 \overline{) 537} \\ 13 \ 8 \ 6 \\ \underline{3 \ 6 \ 14 \ 3} \\ 4 \ 9 \ 6 \\ \text{L. } 24 \ 12 \ 3 \end{array}$	$\begin{array}{r} 40 \overline{) 537} \\ 13 \ 8 \ 6 \\ \underline{4 \ 8 \ 19} \\ 2 \ 4 \ 9 \\ \text{L. } 24 \ 12 \ 3 \end{array}$
--	---	---

<i>Yds.</i>	<i>Yds.</i>	<i>lb.</i>
$\begin{array}{r} 80 \overline{) 878, \text{ at } 3\frac{1}{2}\text{d.}} \\ 10 \ 19 \ 6 \\ \underline{1 \ 16 \ 7} \\ \text{L. } 12 \ 16 \ 1 \end{array}$	$\begin{array}{r} 60 \overline{) 275, \text{ at } 4\frac{1}{2}\text{d.}} \\ 4 \ 11 \ 8 \\ \underline{11 \ 5\frac{1}{2}} \\ 5 \ 8\frac{1}{4} \\ \text{L. } 5 \ 8 \ 10\frac{1}{4} \end{array}$	$\begin{array}{r} 40 \overline{) 440, \text{ at } 7\frac{1}{2}\text{d.}} \\ 11 \\ \underline{4 \ 1 \ 16 \ 8} \\ 9 \ 2 \\ \text{L. } 13 \ 5 \ 10 \end{array}$

P R A C T I C E.

87

<i>Yds.</i>	<i>Yd.</i>	<i>Yds.</i>	<i>Yds.</i>
5)60, at 4s.	96, at 6s.	20)74, at 11s.	20)68, at 17s.
<u> </u>	<u> </u>	<u> </u>	<u> </u>
L. 12	3	5	8
	L. 28 16	37 0	54 8
		<u> </u>	<u> </u>
		3 14	3 8
		<u> </u>	<u> </u>
		L. 40 14	L. 57 16

<i>Yds.</i>	Or thus.	Or thus.
20)260, at 1s. 9d.	260	80)260
<u> </u>	<u> </u>	<u> </u>
2)13	7	3 5
2) 6 10	80)1820	<u> </u>
<u> </u>	<u> </u>	7
3 5	22 15	22 15
<u> </u>		
L. 22 15		

<i>Yds.</i>		
10)260, at 2s. 7d.	80)260	6)260
<u> </u>	<u> </u>	<u> </u>
6} 26	7	5
8} 4 6 8	60)1820	1300
<u> </u>	<u> </u>	<u> </u>
3 5	30 6 8	43 2
<u> </u>	<u> </u>	<u> </u>
L. 33 11 8	3 5	40)1343 2
	<u> </u>	<u> </u>
	33 11 8	33 11 8

10)621 lb. at 3s. 10d.	8} 621 yds. at 3s. 11d.	
<u> </u>	<u> </u>	
2) 62 2 at 2s.	120} 77 12 6 for 2s. 6d.	
2} 31 i at 1s.	2) 38 16 3 for 1s. 3d.	
3} 15 10 6 at os. 6d.	5 3 6 for os. 2d.	
<u> </u>	<u> </u>	
10 7 at os. 4d.	121 12 3 for 3s. 11d.	
<u> </u>	<u> </u>	
119 — 6 at 3s. 10d.		

What

What cost 370 yards broad cloth, at 9s. 6d. the yard?

$$\begin{array}{r} 4 \overline{) 370} \\ 5 \overline{) 92 \ 10} \\ 8 \overline{) 74} \\ 9 \ 5 \end{array}$$

L. 175 15

$$\begin{array}{r} 20 \overline{) 370} \\ 4 \overline{) 148 \ 0} \\ 2 \overline{) 18 \ 10} \\ 9 \ 5 \end{array}$$

175 15

$$\begin{array}{r} 2 \overline{) 370} \\ 40 \overline{) 185} \text{ at 10s.} \\ \text{Sub. } 9 \ 5 \text{ at 6d.} \\ 175 \ 15 \end{array}$$

What cost 212 yards, at 13s. 7d.

$$\begin{array}{r} 4 \overline{) 212} \\ 5 \overline{) 53} \\ 42 \ 8 \\ 8 \overline{) 42 \ 8} \\ 6 \overline{) 5 \ 6} \\ 17 \ 8 \end{array}$$

L. 143 19 8

$$\begin{array}{r} 20 \overline{) 212} \\ 6 \overline{) 127 \ 4} \\ 3 \overline{) 10 \ 12} \\ 6 \overline{) 5 \ 6} \\ 17 \ 8 \end{array}$$

143 19 8

$$\begin{array}{r} 2 \overline{) 212} \\ 10 \overline{) 106} \\ 2 \overline{) 21 \ 4} \\ 2 \overline{) 10 \ 12} \\ 6 \overline{) 5 \ 6} \\ 17 \ 8 \end{array}$$

143 19 8

What cost 450 yards, at 17s. 11d.?

$$\begin{array}{r} 4 \overline{) 450} \\ 112 \ 10 \\ 112 \ 10 \\ 2 \overline{) 112 \ 10} \\ 6 \overline{) 56 \ 5} \\ 9 \ 7 \ 6 \end{array}$$

L. 403 2 6

$$\begin{array}{r} 2 \overline{) 450} \\ 225 \\ 2 \overline{) 112 \ 10} \\ 6 \overline{) 56 \ 5} \\ 9 \ 7 \ 6 \\ 403 \ 2 \ 6 \end{array}$$

$$\begin{array}{r} 20 \overline{) 450} \\ 8 \overline{) 360 \ 0} \\ 2 \overline{) 22 \ 10} \\ 2 \overline{) 11 \ 5} \\ 3 \overline{) 5 \ 12 \ 6} \\ 3 \ 15 \\ 403 \ 2 \ 6 \end{array}$$

What

P R A C T I C E.

89

What cost 132 yards, at 19s. 1d.?

$$\begin{array}{r} 20)132 \\ 9 \end{array}$$

$$118 \ 16$$

$$\begin{array}{r} 12) \ 6 \ 12 \\ 11 \end{array}$$

$$\text{L. } 125 \ 19$$

$$\begin{array}{r} 8)132 \\ 8 \end{array}$$

$$105 \ 12$$

$$\begin{array}{r} 5) \ 16 \ 10 \\ 6) \ 3 \ 6 \\ 11 \end{array}$$

$$125 \ 19$$

$$\begin{array}{r} 60)132 \\ 80 \end{array}$$

$$2 \ 4 \text{ at } 4d.$$

$$2 \ 4 \text{ at } 4d.$$

$$1 \ 13 \text{ at } 3d.$$

$$6 \ 1 \text{ Sub.}$$

$$125 \ 19$$

224, at 4s. 6 $\frac{1}{2}$ d.

$$2$$

$$8)44 \ 16$$

$$12) \ 5 \ 12$$

$$9 \ 4$$

$$\text{L. } 50 \ 17 \ 4$$

60)224, at 4s. 8 $\frac{1}{2}$ d.

$$2$$

$$44 \ 8$$

$$3 \ 14 \ 8$$

$$8) \ 3 \ 14 \ 8$$

$$9 \ 4$$

$$\text{L. } 52 \ 14 \ 8$$

435, at 11s. 7 $\frac{1}{4}$ d.

$$5$$

$$10)217 \ 10$$

$$\begin{array}{r} 3) \\ 4) \end{array} 21 \ 15$$

$$7 \ 5$$

$$\begin{array}{r} 4) \ 5 \ 8 \ 9 \\ 1 \ 7 \ 2\frac{1}{4} \end{array}$$

$$\text{L. } 253 \ 5 \ 11\frac{1}{4}$$

12)435

$$5$$

$$217 \ 10$$

$$36 \ 5$$

From 253 15

$$4) \ 36$$

$$\text{Sub. } 0 \ 9 \ 0\frac{3}{4}$$

$$253 \ 5 \ 11\frac{1}{4}$$

M

4)960

PRACTICE.

4)964, at 5s. 3½d.

$$\begin{array}{r} 20)241 \\ 4)12 \quad 1 \\ \underline{3 \quad - \quad 3} \end{array}$$

L. 256 1 3.

4)964, at 5s. 5¼d.

$$\begin{array}{r} 12)241 \\ 10)20 \quad 1 \quad 8 \\ \underline{2) \quad 2 \quad - \quad 2} \\ \underline{1 \quad - \quad 1} \end{array}$$

L. 264 1 11

What cost 70 acres of land, at L. 5 17, L. 5 15, or L. 5 18, per acre?

2)70, at L. 5 17.

$$\begin{array}{r} 5 \\ \underline{5} \\ 350 \\ 2)35 \\ 5)35 \\ \underline{17 \quad 10} \\ 7 \end{array}$$

L. 409 10

2)70, at L. 5 15.

$$\begin{array}{r} 5 \\ \underline{5} \\ 350 \\ 2)35 \\ \underline{17 \quad 10} \end{array}$$

L. 402 10

2)70, at L. 5 18

$$\begin{array}{r} 5)5 \\ \underline{5} \\ 358 \\ 35 \\ 14 \\ 14 \end{array}$$

L. 413

What cost 60 coach-horses, at L. 15 14 10, L. 18 11 2, or L. 19 18 3 each?

First price.

$$\begin{array}{r} 60 \\ 15 \\ \underline{15} \end{array}$$

20)900

$$\begin{array}{r} 45 \\ \underline{45} \end{array}$$

945

Sub. 10

L. 944 10

Second.

$$\begin{array}{r} 2)60 \\ 18 \\ \underline{18} \end{array}$$

1080

$$\begin{array}{r} 10)30 \\ 6)3 \end{array}$$

10

L. 1113 10

Third.

$$\begin{array}{r} 60 \\ 19 \\ \underline{19} \end{array}$$

1140 at L. 19.

54 at 18s.

15 at 3d.

L. 1194 15

According to a computation made by Mr. EMPSON, in 1761, the cattle slaughtered in London were as follows,

follows, in number, viz. Bulls, oxen, and cows 78254, sheep and lambs 711121, calves 104760, hogs for pork 146932, hogs for bacon 41000, sucking-pigs 52600; Now, valuing the black cattle at L. 5 15, the sheep at 12s. 6d. the calves at 25s. the hogs, for pork, at L. 3 5, those for bacon, at L. 3 15, and the pigs at 9s.; how much did London expend in butchers meat that year?

Answer, L. 1680310. We are far within the value.

§ 2. DECIMAL PRACTICE.

8)750lb. at $\frac{1}{4}$ d. 2)560, at 2d. 60)572, at 5d.

40) 93.75

60)280

4) 9.533

2.383

L. 2.34375

L. 4.66

L. 11.916

Or L. 2 6 10 $\frac{1}{2}$

Or L. 4 13 4.

Or L. 11 18 4

30 } 537, at 11d. 40)440, at 7 $\frac{1}{4}$ d. 2)74, at 11s.

80 }

17.9

6)11

10)37

6.7125

4) 1.8333

3.7

L. 24.6125

.4583

L. 40.7

L. 13.2916

10 } 621, at 3s. 5)224, at 4s. 6 $\frac{1}{4}$ d. 3 } 435, at 11s.

12 }

[10d. 8) 44.8 5 } 435, at 11s.

62.1

8) 44.8

80 }

[7 $\frac{1}{4}$ d.

10) 51.75

12) 5.6

145

5.175

.46

87

L. 119.025

L. 50.86

14.5

5.4375

1.359375

L. 253.296875

CHAP.

C H A P. X.

ALLIGATION MEDIAL.

Quest. 1. An inn-keeper mingles 20 pints of porter, worth 4d. per pint, with 30 pints of cider, at 6d. 12 pints of brandy, at 2s. 6d. I demand what a pint of the mixture is worth? Answer 10d.

2. Bought at Kilbride market 3 parcels of hogs, the first 273, at L. 5 10 each 21; the next 187, at L. 5 15 each 22; the third 194, at 5s. a-head; I put them all into one drove: I demand what they stand a-head, one with another? Answer 5s. 1½d. nearly.

3. There is a mass of 56oz. of gold, at L. 3 18, and silver, at 5s. per ounce; the value of the whole is L. 87: I demand the quantity of gold and silver?

Answer 20 oz. gold, 36 silver.

Multiply the total quantity by one of the rates; subtract this product from the whole value; divide the remainder by the difference of the rates; the quotient is the quantity of the dearest, or cheapest, according as you multiplied by the lower or higher rate.

Several curious questions may be answered by this rule; such as,

4. There are 100 yards cloth; scarlet, at 15s. and green, at 9s. a-yard; value L. 63: What yards of each?

Answer 60 and 40.

5. Twenty gentlemen and ladies, at a tea-table, drunk 93 cups of tea; each gentleman 5, and each lady 4: How many were the gentlemen and ladies?

Answer 13 gentlemen, 7 ladies.

6. There

6. There are 100 gold and silver watches, value L.1200; the gold at L.40, the silver at L.5 each: How many of each metal?

Answer 20 of gold, and 80 of silver.

§ 2. ALLIGATION ALTERNATE.

CASE I.

1. A farmer has corn of 4 kinds, viz. Wheat, at 3s. rye, at 2s. 6d. barley, at 2s. 2d. oats, at 1s. 8d. per bushel; of all these he would make a composition that might be worth 2s. 4d. per bushel: I demand how much of each grain he must use?

$$28 \left\{ \begin{array}{l} 36 \\ 30 \\ 26 \\ 20 \end{array} \right\} \left| \begin{array}{l} 8 \\ 2 \\ 2 \\ 8 \end{array} \right. \quad \text{Or thus:} \quad 28 \left\{ \begin{array}{l} 36 \\ 30 \\ 26 \\ 20 \end{array} \right\} \left| \begin{array}{l} 2 \text{ of wheat.} \\ 8 \text{ of rye.} \\ 8 \text{ of barley,} \\ 2 \text{ of oats.} \end{array} \right.$$

Place the several prices under one another, and the common price on the left; join one greater than the common price with one less, and set the difference betwixt each and the common price opposite to the one joined with it; and how many ways you couple them you have so many answers. The first way they are coupled above you have 8 of wheat and oats, and 2 of rye and barley; and the other way changes the numbers: but they might be coupled both ways at once, and the answer would be 10 of each kind: also multiply or divide these numbers as you please, the products and quotients will still answer the question: or thousands of other answers may be had, as will appear next question.

2. A vintner has wine, at 2s. and brandy, at 18d. which he would mingle with sugared water, worth 1d. the pint, so as the punch might be worth 8d. what quantity must he mix of each?

$$8 \left\{ \begin{array}{l} 24 \\ 18 \\ 1 \end{array} \right\} \left| \begin{array}{l} 7 \\ 7 \\ 16.10 \end{array} \right. \quad \left| \begin{array}{l} 7 \\ 7 \\ 26 \end{array} \right.$$

Answer, 7 of wine, 7 of brandy, and 26 of water. Now make a mixture of two of the simples, suppose of the wine and water you must take 7 pints of wine to 16 of water, as you will find by subtracting the common price from the wine, and the water from the common price: Now add this 7 and 16 to the composition found, you may have 10000 answers, thus:

wine,	7.	14.	21	28	35	42	49	56	63	&c.
Bran.	7.	7.	7.	7.	7.	7.	7	7.	7.	&c.
Wat.	26.	42.	58.	74.	90.	106.	122.	138.	154.	&c.

Or if you make a composition of the brandy and water, you will find 7 of the one to 10 of the other; by which increase as before.

Wine	7	7.	7	7.	7.	7.	7.	7.	&c.
Brandy	7	14	21.	28.	35.	42.	49.	56.	&c.
Water	26.	36.	46.	56.	66.	76.	86.	96.	&c.

But if you make the water invariable, you must increase the wine while you diminish the brandy; or contrariwise, as they are both too dear, and you vary the wine by 10, that is the difference betwixt the brandy and the mixture, and the brandy by 16, the difference of the wine from the mixture; and as the former numbers were small, take others in the same proportion.

Wine	56	46	36	26	16	56	66	76	86.
Brandy	56	72	88	104	120	or 56	40	24	8.
Water	208	208	208	208	208	208	208	208	208.

The sums or differences of any of these trains of answers would answer the purpose in the same manner.

CASE II.

ALLIGATION PARTIAL.

1. A miller hath 20 quarters of fine flour, at 18s. per quarter; which he would mingle with a second fine, at 15s. rye-flour, at 12s. and barley-flour, at 10s. so as to make up a composition worth 13s. 4d. I demand how much of the coarser wheat, rye and barley flour, he must mix with the 20 of fine?

$$40 \left\{ \begin{array}{l} 54 \\ 45 \\ 36 \\ 30 \end{array} \right\} \left| \begin{array}{l} 10 \text{ fine.} \\ 4 \text{ coarse.} \\ 5 \text{ rye.} \\ 14 \text{ barley.} \end{array} \right.$$

These would answer the price; but as there must be 20 of the fine, say, by the Rule of Three, 10:4::20:8 &c. for the others the same way; so you have 8 of the coarse, 10 rye, and 28 barley. It may be coupled otherwise, or varied, as in the former Case; only the 20 of fine flour must be still the same; for instance, make a composition of the coarse, the rye and barley, at the given prices, you will find 14 of the coarse, and 5 of the rye and barley, and these continually.

Wheat	20	20	20	20	20	20	20	20
Wheat	8	22	36	50	64	78	92	106
Rye	10	15	20	25	30	35	40	45
Barley	28	33	38	43	48	53	58	63

CASE III.

ALLIGATION TOTAL.

1. A grocer would mingle currants at 9d. raisins, at 8d. figs, at 5d. and prunes, at 4d. per lb. so as to make a composition worth 6d. and the quantity 60lb.

Couple just as before.

ALLIGATION.

6	{	9		2 currants.
		8		1 raisins.
		5		2 figs.
		4		3 prunes.
				8

The answer should be 60; wherefore say, by the Rule of Three, $8 : 2 :: 60 : 15$. $8 : 2 :: 60 : 15$. $8 : 1 :: 60 : 7.5$ and $8 : 3 :: 60 : 22.5$ which answers the question; and these numbers may be varied by this rule.

Let one of the numbers be invariable, and of the three others let one be of a different kind from two, viz. one greater, and two less than the mixture, or the contrary: then of the two that are of one kind, take their differences from the mixtures, and subtract one difference from the other, and with the remainder either increase or diminish the single variable quantity, then let that quantity that differs most from the mixture, be varied in the same manner, by the sum of the two other differences: and the last variable quantity must be increased or lessened by the sum of the remaining two differences; but in a contrary manner, viz. added when they were subtracted, so will the numbers found be varied thus:

Currents	15	15	15	15
Raisin	7.5	8.5	9.5	10.5
Figs	15	11	7	3
Prunes	22.5	25.5	28.5	31.5

Or contrariwise, thus:

C.	15	15	15	15	15	15	15	15
R.	7.5	6.5	5.5	4.5	3.5	2.5	1.5	.5
F.	15	19	23	27	31	35	39	43
P.	22.5	19.5	16.5	13.5	10.5	7.5	4.5	1.5

Any

Any one may be made invariable, and other answers obtained, and the quantities might be coupled two other ways still.

2. A founder would mix gold, silver, brass, and tin together; gold, at L. 3 14 per ounce, silver, at 5s. brass, at 9d. and tin, at 3d. he would make of the mixture a bell of 200 ounces, that might be worth L. 200: How much of each metal must he take?

Answer 54 oz. of each.

CASE IV.

A question may be proposed that is both total and partial, and which, being a compound of the other two, may be solved as the following:

A farmer has 500 stones of clover, worth 6d. per stone, with plenty of rye-grass, worth 4d. natural grass, at 3d. and oat-straw, at 2d. per stone: Now, he would mix the three last in such quantities with the 500 stones clover, as would make a stack of 4000 stones, and that might be worth $3\frac{1}{2}$ d. per stone; what stones of each must he mingle?

Work first as in Alligation Partial, thus:

$$\begin{array}{r|l|l}
 \begin{array}{l} 12 \\ 8 \\ 6 \\ 4 \end{array} & \begin{array}{l} 1 \ 3 \\ 1 \ 3 \\ 5 \ 1 \\ 5 \ 1 \end{array} & \begin{array}{l} 4 \\ 4 \\ 6 \\ 6 \end{array} \\
 \begin{array}{l} \text{Then } 4:4::500:500 \text{ ryegrass.} \\ 4:6::500:750 \text{ grass.} \\ 4:6::500:750 \text{ straw.} \end{array} & &
 \end{array}$$

20

These quantities, with the clover, make 2500 stones, at the given price; but there should be 4000: wherefore make a composition of 1500 stones of the three last kinds, at the given price, you will find 1000 of rye-grass, 250 of hay and straw; so you have 1500 of rye-grass,

N

grass,

grafs, 1000 of common hay, 1000 of straw. Many varieties may be found, as before, thus :

Clover	500	500	500	500	500	500
Ryegrass	1500	1498	1496	1494	1492	1490
Hay	1000	1004	1008	1012	1016	1020
Straw	1000	998	996	994	992	990

Promiscuous Examples.

1. A cook sends his servant to buy 20 fowls, of five different kinds and prices ; but gives him only 20d. to pay for the whole. Now, he was to buy geese, at 4d. hens, at 2d. chickens, at 1d. doves, at $\frac{1}{2}$ d. and larks, $\frac{1}{4}$ d. How many of each kind must he buy ?

See ten different answers.

Geese	2	2	2	2	2	1	1	3	2	1
Hens	2	1	2	2	2	3	1	1	1	2
Chickens	4	4	5	3	2	6	12	2	7	9
Doves	4	11	1	7	10	6	2	2	2	4
Larks	8	2	10	6	4	4	4	12	8	4

To find one of these answers, leave out the chickens, make a composition of the geese and larks, another of the hens and doves, at the common prices, diminish their quantities, that, when they are added, they may be under 20, and make up 20 with the chickens.

2. There is a flock of geese, ducks and hens, 60 in number, value L. 3, and weight 23 stones ; I demand the number of each, a goose being 18d. a duck a shilling, and a hen 8d. value ; also a goose weighs 10lb. a duck 6lb. and a hen 4lb. ?

Answer 8 geese, 40 ducks, 12 hens.

The common price being the same with that of a duck, make a composition of the geese and hens, you will

will find 2 geese and 3 hens; put these together; the 5 will weigh 32 lb. so their common weight is 6.4; couple this with the ducks which are 6 lb. weight, and their differences, from the common weight, you find .13 and .26, which added make .4 which should be 60: then $.4 : .13 :: 60 : 20$ and $.4 : .26 :: 60 : 40$ the number of ducks; of the 20 there must be 2 geese for 3 hens: therefore $5 : 20 :: 3 : 12$ hens, &c.

3. A man owed me L. 100, with 4 years interest, at 5 per cent. and going to receive payment, he had only moidores, guineas, and 18s. pieces; and when I doubted what we would do for change, he said he would pay me with 120 of these pieces: How many of each think you I would receive?

Answers are innumerable.

Moidores	20.	17.	23.	26.	14.	11.	8.	5.	18.	&c.
Guineas	20.	29.	11.	2.	36.	47.	56.	60.	26.	&c.
18s. pieces	80.	74.	86.	92.	68.	62.	56.	50.	76.	&c.

4. There is L. 1800 in 18000 pieces, viz. guineas and half-guineas, equal in number, crowns and half-crowns, ditto, shillings and six-pences, ditto: I demand the number of each?

Answer 125 guineas and half-ditto, 3125 crowns and half-ditto, 5750 shillings and six-pences.

CHAP. XI.

TARE AND TRET.

Quest. 1. What is the neat weight of 20 hhds. of sugar, each weighing, gross, 12 cwt. 3 qrs, tare, 94 lb. per hoghead? Answer 238 cwt. 24 lb.

2. What

2. What is the neat weight of 14 cwt. 3 qrs. 10 lb. sugar, tare 12 lb. per cwt. ? Answer 13 cwt. 1 qr.

3. In a chest of sugar, weighing gross 18 cwt. 2 qrs. 20 lb. tare 2 cwt. and tret 4 lb. on 104 lb. What is the neat ? Answer 16 cwt. 4 lb.

Subtract the tare, leaves the futtle ; from the futtle take $\frac{1}{16}$ part.

4. In 16 hhds. sugar, each weighing 12 cwt. 3 qrs. tare 62 lb. per hoghead, and tret 4 on 104, as usual ; What is the neat ?

Answer 187 cwt. 2 qrs. 15 $\frac{1}{2}$ lb.

5. The neat weight of a chest of sugar 16 cwt. 4 lb. the allowance of tare was 20 lb. and tret as usual ; What was the gross weight ?

Answer 16 cwt. 3 qrs. 11 lb. nearly

Note, The 25th part of the neat is equal to the 26th part of the futtle ; and so must be the tret ; and this, added to the neat, gives the futtle.

6. A large cargo of tea has allowance of 7 cwt. 3 qrs. tare, and the tret, computed as usual, comes to 4 cwt. 1 qr. I demand both the gross and neat weights ?

Answer 118 cwt. 1 qr. gross, 106 cwt. 1 qr. neat.

The tret multiplied by 25, gives the neat ; or by 26, gives the futtle.

7. The gross weight of a cargo of bale goods is 17 cwt. 1 qr. 12 lb. and the neat weight is 16 cwt. 2 qrs. What is the tare and the tret ?

Answer Tare 22.08 lb. tret 2 qrs. 17.92 lb.

8. There are 2 hhds. of sugar, weighing together 23 cwt. 24 lb. gross, 20 cwt. 3 qrs. 26 lb. neat ; the allowance of tare on the greater is double that on the lesser, and its tret exceeds that of the other 8 lb. I demand

mand both the gros and neat weight of each hogf-head?

Anfw. The greater 12cwt. 3qrs. 2lb. gros, 11cwt. 1qr. 15lb. neat; the leffer 10cwt. 1qr. 22lb. gros, 9cwt. 2qrs. 11lb. neat.

The 8lb. difference of tret multiplied by 26, gives the difference of the futtle; the 26th part taken from that, leaves the difference of the neat; the rest is easy: for half the difference of the neat added to half the sum of the neat, given in the question, gives the greater quantity, or subtracted, leaves the leffer.

9. Five hogfheads tobacco, five thousand pounds neat,

When allowance of tare was the same with the tret;

Be pleas'd to resolve me, I am at a loss,
To know what they weigh'd upon the quay,
gros.

Answer 5400 lb. gros.

10. In nineteen tons gros weight, say, What is the neat,

When tare is exactly the same with the tret?

Answer, 17 tons, 11cwt. 3qrs. $11\frac{1}{27}$ lb.

Here 27 gros would give 25 neat: fo

$$27 : 25 :: 19 : 17\frac{3}{27}.$$

11. A ship's load of tobacco, weighing gros 500 tons, is offer'd me at L. 5 per cwt. neat, tare being allowed at 2 lb. per cwt. and tret as usual; or, I may have it at L. 4 14 6. per cwt. gros: Which is the best offer? Answer, The first, by L. 31 11 3.

12. Bought a chest of sugar, weighing gros 2000lb. at 6d. the lb. neat, the whole cost L. 46 5; but going back to buy another, I got 26 lb. less allowance of tare, and that chest stood me L. 46 17 6: What tare

was

was I allowed on either chest, tret on both being as usual? Answer 76lb. the first, 50lb. the last chest.

The difference of the amounts will give the difference of pounds neat.

13. Bought two boxes of tea, each weighing gross 676lb. for L.428 15, the tare allowed on the first, was double that on the second, and the tret was as 96 to 100; What did I pay for $\pounds$ pound neat?

Answer 7s.

The proportion of the tret being as 96 to 100, the same is the proportion of the futtle: Now, add to 96 and 100 such two numbers (whereof one is double the other) as will make them equal, viz. add 8 to 96, and 4 to 100, makes each of them 104: then say, 104 : 8 :: 676 : 52 the tare on the first box, and 104 : 4 :: 676 : 26 that of the second, the rest is easy.

14. There is a parcel of tea, which, valued at 7s. 6d. the lb. gross, will amount to L.84, the futtle at the same rate, amounts to L.78, and the neat to L.75: What is the weight of the tea?

Answer 224lb. gross, 200 lb. neat.

15. I bought some barrels Orkney butter,
To put aboard a smuggling cutter;
The pound, if paid at six-pence gross,
Shall make me ten per cent. of loss;
By selling out at seven-pence neat:
Now, what allowance, tare and tret,
Shall I propose at payment of it,
To save me ten per cent. of profit?

Ans^r. There must be allowed $\frac{1}{10}$ of the gross; $\frac{1}{10}$ of which may be considered as tare, and $\frac{1}{10}$ as tret; so that the neat to be paid for is $\frac{8}{10}$ of the gross, and it holds out in the sale at $\frac{9}{10}$ of the gross.—We leave the operation, and the rule to exercise the learner, and whet the edge of his understanding.

§ 2. LOSS

§ 2. LOSS AND GAIN.

1. Bought 240 stones of butter, at 8s. 6d. per stone; how must I sell it again, so as to make L.5 by the bargain? Answer, At 8s. 11d.

2. Bought 960 stones wool, at 7s. 6d. the stone; which I sold at 7s. 8d. What did I gain by the bargain? Answer L.8.

Thus, $120)960(8$

3. Bought cloth, at 15s. per yard; how must I sell it again, to make L.10 on the L.100?

Answer, At 16s. 6d.

4. Bought cows, at L.3 12 each; how must I sell them, so as to make 8 per cent.

Answer L.3 17 $9\frac{3}{4}$.

5. Bought a tun of wine for L.25, and sold it for L.27; what did I gain per cent?

Answer 8 per cent.

6. Bought a pair of buckskin-breeches, for 22s. 6d. which I sold for 24s. What did I gain per cent?

Answer L. 6 13.

7. In selling cloth, at 17s. 6d. per yard, I gained 10 per cent. what would my gain be, when I raised my price to 18s.? Answer $13\frac{1}{7}$.

Say thus: $35 : 110 :: 36 : 113\frac{1}{7}$ and subtract 100.

8. In selling raisins, at 8d. the lb. I gained 12 per cent. but afterward was obliged to sell them at 6d. Whether did I gain or lose then, and what per cent?

Answer, Lost 16 per cent.

$8 : 112 :: 6 : 84$ take from 100.

9. In selling yarn at 4s. per spindle, gained 9 per cent. what did the yarn cost me? Answ. 3s. 8 $\frac{1}{4}$ d.

$109 : 4 :: 100 :$

10. Selling

10. Selling cotton, at 9d. per lb. I lost 4 per cent. what did it cost me? Answer $9\frac{1}{8}$ d.

$$96 : 9 :: 100$$

11. Bought linen, at 2s. 6d. which I sold at 3s. per yard, and gave 3 months credit: What did I gain per cent.? Answer $18\frac{1}{2}$ per cent. or rather more.

See Simple Interest, viz. find the present worth of 3s. due 3 months hence.

12. Bought cambrics, at 5s. a-yard, what must I sell it for, and give 6 months credit, so as to gain 12 per cent.? Answer 5s. 8d. $3\frac{1}{2}$ qrs.

$100 : 112 :: 5 : 5.6$ and find the amount of that for 6 months.

13. Selling cloth at 9d. the yard profit, I gained 8 per cent. what did the cloth cost me?
Answ. 9s. $4\frac{1}{2}$ d.

8)100(12.5 nine-pences. The gain per cent. must be as often contained in 100 as the gain on a yard in the cost of a yard.

14. A merchant selling fustians at 11s. 6d. gained 12 per cent. and afterwards raised them to 12s. what did he gain by this last sale? Answer $16\frac{10}{11}$ per cent.

$23 : 112 :: 24$ This question is wrong in HILL's *Arithmetic*.

15. Bought broad cloth, at 12s. and had 9 months credit, and sold it at 15s. and gave 12 months credit: What did I gain per cent.? Answ. L.23 15 5.

Find the present worth of 12s. due 9 months hence, and of 15s. due 12 months hence, and consider these as your real prices.—See *Simple Interest*.

16. Bought

16. Bought sugar, at 50s. per cwt. which, after it had lain by me a full year, I sold for 54s. and gave a year's credit, though I paid ready money myself: What did I gain per cent.? Answer 1.66.

Add one year's interest of 50s. to itself, consider this as the buying-price; take a-year's discount of 54s. from it, call the remainder the selling-price: The gain on the cwt. is 10d. $50 : 10 :: 100 \text{ \&c.}$

17. Selling cloth the lowest in the market, I gained 5 per cent. but afterwards raising my price 6d. I gained 10 per cent. What did I pay for the cloth myself? Answer 10s.

Your 6d. difference of gains makes L. 5 more gain: Therefore $5)100(20$ six-pences.

18. Sold buckskin breeches, at 8 per cent. gain; but finding them too cheap, I raised them 1s. each pair, and then gained 12 per cent. What did they stand me? Answer 25s.

$4)100(25$ for as 1s. is the same gain on the pair, that L. 4 is on L. 100, L. 4 must be as often contained in 100 as 1s. in the price of the pair.

19. I overheard two dealers communing upon a bargain of cloth, but heard no price mentioned, only the buyer said, He had offered enough; for at that rate the seller would have 1s. on the pound gain: but the seller insisted on 1s. more; For, says he, a penny on the shilling is moderate enough profit: I demand how much the cloth had cost him, how much he demanded for it, and what the other had bidden him?

Answer, It cost him 30s. a-yard; he would sell it at 32s. 6d. was bidden 31s. 6d.

A shilling on the pound is 5 per cent. a penny on the shilling is $8\frac{1}{3}$ per cent. the difference is $3\frac{1}{3}$: So the

the question is wrought as the last, if you please; or rather thus: $20 : 21 :: 12 : 12.6$ and $13 - 12.6 = .4$ 12(308.

20. I bought herrings, at 6s. 8d. per 100; how many may I sell for a penny, to gain 25 per cent.?

Answer, Only 1.

21. I have very good cheese, which cost me 5s. per stone; for which I am offered 5s. 4d. ready money, or 5s. 6d. at 6 months: which offer is the best, and what is the gain per cent. both ways?

Answer, The gain by the first is $6\frac{2}{3}$, by the last 7.317.

22. Bought some brandy, at 2s. 8d. the Scotch pint; which proving but bad, I would be glad to sell with the loss of L.10 for every L.100 I recover: What must I sell a pint for? Answer 2s. 5d.

23. Laid out some hundred pounds upon tobacco, at L.9 per cwt. which, being damnified, I am willing to lose 10 per cent. by it: For what must I sell the cwt.? Answer L.8 2.

24. My neighbour and myself bought each of us 100 cattle, which cost L.700 betwixt us: In the sale, he made of his 10 per cent. and I made 12s. on each beast: Our gains together came to be L.100: I demand each man's part of the gain, also what he gained on the beast, and my own gain per cent.?

Answer, Your gain L.60, his L.40; he gained 8s. on the beast, you 20 per cent.

25. A merchant is offered goods, at 11s. 3d. which he can sell again at 12s. also goods at 17s. 8d. which he can sell at 18s. 10d. Which will he gain most upon, and what per cent.?

Answer, By the first he gains L.6 13 4, by the last L.6 12 $\frac{1}{2}$.

26. Bought 250 yards of scarlet cloth, at 15s. 4d. and got a small piece to the bargain; sold half my whole quantity for L.100, and the rest at 16s. a-yard, and

and find I have made 12 per cent. profit : How many yards were in the small piece ? Answer $36\frac{2}{3}$.

27. A butcher buys an ox, a hog and a wedder, for L.31 5 : In the sale he gains as much on the ox as he paid for the hog, and that was 20 per cent. and as much on the hog as he paid for the wedder, which was more than 20 per cent. and though he sold the wedder at the same rate he bought him for, yet he made 20 per cent. on the whole : I demand what each cost him ?

Answer, The ox L.25, the hog L.5, and the wedder 25s.

28. Bought a quantity of holland, at 4s. 2d. per ell Flemish ; for which I am offered 5s. 6d. per yard English ; Whether shall I gain or lose by accepting this offer ?

Answer, Lost $\frac{1}{2}$ d. on the ell Flemish, or $\frac{2}{3}$ on the yard English.

29. Bought a quantity of holland, at 3s. 9d. per ell Flemish ; and by selling it again, at 6s. $3\frac{1}{3}$ d. per ell English, I gained L.5 on the whole : I demand what was the quantity ? Answer 1800 ells English.

30. A farmer is offered 30s. per boll for his barley, ready money, or he is offered 31s. 6d. if he will give a peck to the boll, and 3 months credit : Which offer is the best ? Answer, The first by $8\frac{3}{4}$ d.

31. Bought 40 stones of butter, at 9s. per stone ; also 60 stones ditto, at 9s. 6d. putting these two quantities together, I sold all at 9s. 4d. I demand whether I gained or lost, and how much ?

Answer, Gained 3s. 4d. only.

32. A parcel of goods that lay long in my custody, And cost me six hundred pounds seven years ago,

I sold for a thousand, and gave seven years trust to-day :

How much have I gained or lost, I would know ?

Answer, Reckoning simple interest at 5 per cent.
you

you lose L. 69.259, but at 4 per cent. you gain L. 13.25; or finding the amount of L. 600 for 14 years, and taking L. 1000 from it, the loss is L. 20; or, at compound interest L. 187 19 2.

CHAP. XII.

B A R T E R, &c.

Quest. 1. Two merchants, viz. A and B barter; A has 400 yards broad cloth, at 16s. per yard, B has duffle, at 6s. 6d. per yard: It is demanded how many yards duffle B must give A for his cloth?

Answer, 98 $\frac{1}{4}$ yards.

2. Two barter; A has 7cwt. of snuff, at L. 5 per cwt. for which he takes value of B, $\frac{2}{3}$ in cotton, at 15d. $\frac{1}{3}$ in hemp, at 7d. and $\frac{1}{3}$ in flax, at 1s. per lb. How many lb. of each will he receive for his snuff?

Answer 186 $\frac{2}{3}$ lb. cotton, 400 hemp, 333 $\frac{1}{3}$ of flax.

3. Two barter; A has 700 lb. of silk, at 45s. per lb. for which B gives him 18cwt. 3qrs. of cinnamon: I demand how he rated a pound of that cinnamon?

Answer, At 15s.

4. Two barter; A has silver buckles, worth 15s. the pair, but in barter he will have 16s. B has thread stockings, worth 2s. 6d. a-pair, ready money: How must he rate them in barter, to make his gain equal to that of A? Answer, At 2s. 8d.

5. Two barter; A has fine lawns, at 18s. per yard, which he puts off, at 21s. for which B gave him ribband, at 10 $\frac{1}{2}$ d. per yard: I demand what it was worth in ready money? Answer 9d.

6. A has silk stockings, worth 10s. 6d. the pair; which he barter with B, at 12s. taking value in hats, at 15s. each, which are worth but 13s. 4d. Which of the two is the gainer?

Answer, A gains $\frac{1}{8}$ of his ready-money price, B but $\frac{1}{8}$; therefore A gains.

7. A and B barter; A has 100 yards velvet, at 19s. per yard, B shalloon, at 1s. 6d. and buckram, at 1s. per yard: How much shalloon and buckram must A have for his velvet, so as to receive always 5 yards of shalloon for 2 of buckram?

Answer 1000 yards shalloon, and 400 of buckram.

8. Two barter; A hath 15 cwt. of cheese, worth 40s. per cwt. ready money; but barter it with B, at 42s. taking value in wool, at 7s. per stone, which is worth but 6s. 6d. ready money: I demand how much wool must be given for the cheese, and who is the gainer by the barter?

Answer 90 stones wool, and B gains 15s.

9. There is a tenement of house and lands,
Ten thousand pounds in value as it stands,
Is sold just now, at 10 per cent. advance,
For value due in sheep, at two years hence;
At ten and six-pence each, which more than ten,
Will ne'er produce for all the sons of men:
Now, in your sound opinion, tell me, which
Has here the better bargain, and how much?

Ans. The buyer has the advantage by L.476 5 9 $\frac{1}{2}$ at striking the bargain.

FACTORAGE.

1. What is the commission upon L.570 worth of goods, at 3 $\frac{1}{2}$ per cent.? Answer L.19 19.

2. What is the commission on L.456 6 6, at L.7 7 per L.100? Answer L.33 10 9 $\frac{1}{2}$.

3. What

3. What comes a factor's commission to upon the sale of L. 1465 17 worth of goods, at L. 3 15 6 per cent.? Answer L. 55 6 8 $\frac{1}{2}$.

4. A factor has in his hand belonging to his employer, L. 993 4, which he is to remit to him in goods, and retain 4 per cent. commission: What sum will he lay out to be remitted? Answer L. 955.

104 : 100 :: 993.2 : 955 or, 104 :: 993.2 : 38.2 commission.

5. The gross proceeds of a cargo of goods disposed of by a factor in Jamaica, amount to L. 1146 12, from which he is to deduce 5 per cent. commission; and purchase goods for his employer, and again retain 4 per cent. on returns: Now, what will be the commission, both on sales and purchases, and what will be remitted neat to the employer?

Answer L. 54 12 on sales, L. 42 on returns, L. 1050 to be remitted neat.

6. A merchant at London consigns to his factor in Jamaica, 32 bales of cloth, with orders to dispose of it at L. 172 5 per bale, and to make remittance for value, one half in sugar, at 40s. per cwt. and the other half in rum, at 3s. 4d. per gallon, after retaining commission, at 6 per cent. on sales, and 4 per cent. on returns: I demand what quantity of rum and sugar must be returned by the factor, and how the merchant must sell an hundred weight of the one, and a gallon of the other, to gain L. 1000 beyond the neat proceeds of his cloth?

Answer 1250 cwt. sugar, 15000 gal. rum, to be sold, the sugar at 48s. the rum at 4s.

7. Selling goods for my constituent 10 per cent. above his order, I saved him my own commission: What had I per cent.? Answer 9 $\frac{1}{11}$.

110 : 10 :: 100 : 9.09.

8. Selling

8. Selling goods for my employer 5 per cent. above his invoice price, I gained my commission at $3\frac{1}{2}$ per cent. and 115.8 besides: What was the value of the cargo? Answer L.2400 by the invoice.

$100 : 3.5 :: 105 : 3.675$ and $105 - 3.675 = 101.325$
his neat proceeds on L.100, at the advanced price;
which, at the invoice price, would have been 96.5 :
so that on L.100 the advantage is 4.825 : Then say,
 $4.825 : 100 :: 115.8 : 2400$.

INSURANCE.

1. Insured to the value of L.1200, at $12\frac{1}{2}$ per cent. what will the premium amount to? Answ. L.150.

2. Having insured L.1500, what will I recover, in case of loss, discount at 2 per cent.? Answ. L.1470.

3. What sum must I insure, that I may recover L.588, in case of loss, discount at 2 per cent.?

Answer L.600.

$$98 : 100 :: 588$$

4. What sum must I insure, to cover L.1740, premium 10 per cent. and 3 per cent. discount, in case of loss? Answer L.2000.

$$10 + 3 = 13 \quad 100 - 13 = 87 : 100 :: 1740$$

To prove this, add the premium to the value of the cargo, and subtract the discount from the sum insured, the sum and remainder are equal.

5. What is the amount of the insurance of a voyage out and home, at 9 per cent. the cargo L.730 value? Answer L.137 6 3.

The insurance outward is L.65 14, add this to L.730, makes L.795.7; the insurance on that sum is L.71.613, to this add L.65.7.

6. What

6. What will be the amount of insurance out and home, to cover L.720, at 8 per cent. premium, and 2 per cent. discount in case of loss?

Answer, L.133 13 9.

As in Exam. 4th, say, $90 : 100 :: 720 : L.800$ to be insured outward, the premium of which is L.64; which add to the value of the cargo; and again say, $90 : 100 :: 784 : L.871.11$ to be insured home, the premium on which is 69.68 and $69.68 + 64 = 133.68$; or otherwise, thus: Subtract the premium from the short recovery, and say, $90 : 8 :: 720 : 64$ add this, as before, $90 : 8 :: 784 : 69.68$.

7. Underwrote to John Cairns L.500, at 7 per cent. to William Drew L.700, at 11 per cent. to Charles Grant L.800, at 9 per cent. to Matthew Blair L.1000, at 20 per cent. to Robert Agnew L.1200, at 15 per cent. and to Thomas Rolly L.2000, at 12 per cent. received the premiums, and 2 per cent. allowed in case of loss: Now, what shall I suffer by these insurances, supposing the two first to be lost?

Answer L.372.

8. Premium L.128 covered property L.1600, discount 2 per cent. what is the real value of the cargo, and rate of insurance?

Answer 1440 real value, insurance 8 per cent.

$100 : 98 :: 1600 : 1568 - 128 = 1440$

and $1600 : 128 :: 100 : 8$

9. Underwrote to Mr. Jones L.2000, at 9 per cent. and at the same time insured with him L.1800, at 5 per cent. the premiums are passed to account: Now, supposing loss on both sides, what sum will clear us, discount at 2 per cent.

Answer L.106 due to Mr. Jones.

10. Premium L.22 1, short recovery L.240 2, discount 2 per cent. What was the sum insured.

Answer L.245, at 9 per cent.

C H A P. XIII.

E X C H A N G E.

Quest. 1. Glasgow remits to Ireland L.500 Sterling, for how much Irish ought Glasgow to be credited, or how much debt will this pay in Ireland, exchange being at $8\frac{1}{2}$ per cent. or L.100 Sterling equal to L.108 12 6 Irish? Answer L.543 2 6.

2. Dublin is indebted to London in L.700 Irish, for how much Sterling may London value upon Dublin, when exchange is at $10\frac{1}{2}$ per cent.?

Answer L.633 9 8 Sterling.

3. Glasgow remitted to Ireland L.1000 Sterling, when the exchange was at 10 per cent. afterwards exchange rising to 12 per cent. Glasgow draws back the same remittance: What Sterling will be drawn back?

Answer L.982 $\frac{1}{2}$.

110 : 1000 :: 112 Inverse.

4. When exchange was at 15 per cent. Glasgow remitted to Ireland L.1000; but afterwards exchange falling to 12 per cent. Glasgow draws for a reimbursement: For how much Sterling does Glasgow value upon Ireland? Answer L.1026 $\frac{1}{4}$.

115 : 1000 :: 112 Inverse.

5. When exchange was at 14 per cent. I remitted to Ireland L.1581 15; afterwards, on a fall of the rate or course, I drew upon Ireland and received L.1624 10: I demand what was the course of exchange then? Answer 11 per cent.

1581.75 : 114 :: 1624.5 Inverse.

P

6. The

6. The neat proceeds of a cargo of goods from Britain amount in Virginia to L.1465 currency; how much is that in Sterling money, when exchange is at 60 per cent.? Answer L.915 12 6.

7. Jamaica remits to Glasgow the neat proceeds of a cargo, L.416 Sterling; how much currency did the goods amount to, exchange 80 per cent.?

Answer L.748 16.

8. A factor in Virginia having sold his employer's goods, at L.469 2 6 currency, deduces 4 per cent. commission, and remits in full of neat proceeds L.347 10 Sterling: I demand the rate of exchange in this case? Answer L.29 16 2.

104 : 100 :: 469.125 : the currency to be sent home, and 451.08 : 347.5 :: 100 : 129 &c.

9. A parcel of goods from Britain, amounting, by invoice, to L.323 5 Sterling, are sold by a factor in Virginia, for L.469 2 6 currency: What Sterling ought the factor to remit, deducting 5 per cent. for commission, and how much does the employer gain per cent. when exchange is at 30 per cent.?

Answer L.343.681 Sterling to be returned, and the gain L.6 6 $4\frac{1}{4}$ per cent.

Say, 105 : 100 :: 469.125 : 446.785 the currency to be remitted, and 130 : 100 :: 446.785 : 343.681 the Sterling; so the gain on the whole is 20.43131, and the gain per cent. is easily found.—This question is wretchedly blundered by some late authors.

Examples follow of Exchange with Holland, France, and other foreign places, where the money and coins are different. Some account of these coins must be given here.

In Holland, 8 pennings, or 2 duytes, make 1 groat or penny, 2 groats 1 stiver, 6 stivers, or 12 pence, 1 shilling, 20 shillings 1 pound Flemish, 20 stivers, or 40d. 1 guilder or florin, 6 guilders 1 pound Flemish, $2\frac{1}{2}$ guilders

$2\frac{1}{2}$ guilders 1 rixdollar. L. 1 16 7 Flemish the par of the pound Sterling.

In Hamburgh, 12 phennings make 1 schilling-lub, 16 schilling-lubs 1 mark, 2 marks 1 dollar, 3 marks 1 rixdollar, $6\frac{1}{4}$ marks 1 ducat. The ducat is 9s. $4\frac{1}{2}$ d. Sterling. In both these places there is a difference betwixt bank money and current money, which is called the *agio*.

In France, 12 deniers make 1 sol, 20 sols 1 livre, 3 livres, or 60 sols, 1 crown or ecu. Value 2s. $5\frac{1}{4}$ d. Ster.

In Spain, 34 mervadies make 1 rial, 8 rials 1 piaſtre, worth 43d. Sterling; 375 mervadies 1 ducat.

In Portugal, Accounts are kept in rees and milrees; 1000 rees make 1 milree, equal to 5s. $7\frac{1}{2}$ d.

In Venice, 12 deniers d'or make 1 ſol d'or, 20 ſols d'or 1 ducat, equal to 4s. 4d.

In Genoa, 12 denari make 1 ſoldi, 20 ſoldi 1 pezzo or piaſtre, about 4s. 6d. Sterling. There is here lire-money, which is but about $\frac{2}{3}$ or $\frac{1}{6}$ of the exchange-money, and the ſame at Leghorn.

Examples of Exchange with Holland.

1. How many pounds, ſhillings, and pence Flemiſh, are equal to L. 500 Sterling, when exchange is 34s. 8d. Flemiſh per pound Sterling?

Answer L. 866 13 4 Flemiſh.

2. London draws upon Amſterdam for 1296 guilders; how many Flemiſh pounds will London be debited for at Amſterdam, when exchange is at 35s. 6d. per pound Sterling, and how much Sterling pays the draught? Anſw. L. 216 Flemiſh, L. 121.69 Sterling.

6)1296(L. 216 Flemiſh.

3. How many current guilders are equal to 460 bank guilders, when the *agio* is $3\frac{1}{2}$ per cent.?

Answer 476.1.

100 : 103.5 :: 460 :

4. Amsterdam is indebted to Glasgow in L.800 17 Sterling, exchange being at 36s. 6d. bank money: How much currency may Glasgow draw for, when the agio is $4\frac{1}{4}$ per cent.?

Answer L.1523 13 4 current.

$$\begin{array}{cccc} \text{St.} & \text{B.} & \text{St.} & \text{B.} \\ 1 & : 1.825 & :: & 800.85 : 1461.55125 \end{array}$$

$$\begin{array}{cccc} \text{B.} & \text{C.} & \text{B.} & \text{C.} \\ \text{And, } 100 & : 104.25 & :: & 1461.55125 : 1523.667 \end{array}$$

5. Amsterdam is indebted to London L.1484 Sterling: For how many guilders will London draw, when exchange is at 37s. 2d. Flemish per pound Sterling? Answer 16546 $\frac{1}{2}$ guilders.

37.16s. Flemish multiplied by .3, gives the guilders in a pound Sterling.

6. Amsterdam draws upon London for 1470 guilders 6 stivers: For how much Sterling will Amsterdam be debited, when exchange is at 36s. 6d. per pound Sterling? Answ. L.134 5 5 $\frac{1}{2}$ Sterling.

7. A bill of exchange of 12600 guilders Flemish bank money, was purchased at London for L.1200 Sterling; and being remitted to Amsterdam, discharged a debt of L.2205 Flemish current: I demand what was the agio, and what the course of exchange?

Answ. Agio 5 per cent. exchange 35s.

6)12600(L.2100 Flemish.

$$2100 : 2205 :: 100 : \text{agio. } 12 : 21 :: 1 : \text{exchange.}$$

8. What is the course of exchange betwixt Britain and Holland, when the bank guilder is worth 22 $\frac{1}{2}$ d. Sterling? Answer 35s. 6 $\frac{1}{2}$ d.

9. Amsterdam draws upon Paris for 1440 French crowns, exchange 54 $\frac{1}{2}$ d. Flemish per crown: For how many pounds Flemish will Amsterdam be debited at Paris? Answer L.327 Flemish.

10. How

10. How many ducats of Spain will discharge a debt of 4410 bank guilders of Holland, exchange 98d. Flemish per ducat? Answer 1800 ducats.

Examples with Hamburg.

1. London draws on Hamburg for L.680 Sterl. How many marks will clear the draught, exchange 35s. 4d.? Answer 9010 marks.

Note, The mark is 32d. Flemish, and $7\frac{1}{2}$ marks the pound, or the schilling-lub is 2d.

2. Hamburg remits to London a bill of 1982 marks, 13s. lubs: How much Sterling will this clear, when exchange is at 35s. 3d.? Answer L.150.

3. What is the Sterling value of a mark lubs, when the exchange betwixt Britain and Hamburg is 35 $\frac{1}{2}$ s.? Answer, 18d.

4. When exchange was at 37s. Britain remitted to Hamburg a bill of 11655 marks, and upon the fall of the exchange to 35s. Britain drew for the same number: What was the value in Sterling, both of the remittance, and of the draught, and where did the advantage lie?

Ans. L.420 remitted, and L.444 Sterling drawn, so Britain was the gainer.

5. When exchange was at 34s. per pound, Britain drew upon Hamburg for L.612 Sterling, and again, upon the exchange rising to 36s. Britain returned the same in a remittance to Hamburg, to pay her debt: What did Britain gain by this transaction?

Answer L.34.

34 : 612 :: 36 : Inverse.

Examples with France.

1. London draws upon Paris for L.158 17 4 Sterling: How many livres will clear this draught, when exchange is at 32d. per crown? Answer 3574-5.

d. l. d.
32 : 3 :: 38128

2. Londondraws upon Paris for 1512 crowns of exchange: How much Sterling will purchase this draught, when exchange is at $31\frac{1}{2}$ d. per ecu?

Answer L.198 9.

3. A banker in Paris purchases a bill of exchange upon London for 1878 crowns, 6 sols; which being remitted to London, clears a debt of L.234 15 3:1 demand what was the rate of exchange?

Answer 2s. 6d.

4. A banker at London remits to Paris, when exchange is at 30d. Sterling per crown: At what rate must the exchange be, when he draws again, so as to make 5 per cent. profit? Answer $31\frac{1}{4}$ d.

5. How many French crowns are equal to $3612\frac{1}{2}$ ducats of Naples; when 100 of the former are equal to $92\frac{1}{2}$ of the latter? Answer 5000.

Examples with Spain.

1. Madrid is indebted to London L.159 7 6, how many piastres may London value for upon the Spaniard, exchange $38\frac{1}{2}$ d. per piastre?

Answer 1000 piastres.

2. Cadiz remits to London 353.7 piastres, 6 rials, at $40\frac{1}{8}$ d. per piastre: What Sterling will this remittance amount to? Answer, L.602 10 $5\frac{1}{8}$ d.

3. When I am creditor in Spain L.500 8 5, how many piastres may I draw for, exchange $40\frac{1}{2}$ d. per piastre? Answer 2894.

4. In 3264 rials vellon, how many rials of exchange? Answer 1734.

Say, As 32 to 17 The silver is called *plate*, the copper *vellon*.

Examples with Portugal.

1. Britain remits to Lisbon L.1157 19, exchange at 5s. 3d. per milree: What will be received at Lisbon? Answer 4411 milrees, 238 rees.

2. Oporto

2. Oporto draws upon Britain for 1654.320 milrees, exchange $63\frac{1}{8}$ d. per milree: How much Sterling will pay this draught?

Answer L. 436 16 10 2.12.

3. How many piaſtres of Spain are equal to 825552 rees of Portugal, exchange 637 rees per piaſtre?

Answer 1296 piaſtres.

4. How many milrees of Portugal are equal to 54 crowns of Palermo, 889 rees per crown?

Answer 480.060 milrees, or 480 milrees 60 rees.

Examples with Venice.

1. How much Sterling is equal to 1459 ducats, 18 ſols, 1 denier, banco delgiro, at $52\frac{1}{4}$ d. per ducat?

Answer, 320 17 $5\frac{1}{2}$ d.

2. Venice remits to London L. 580 10 Sterling: How many ducats will this debit London in at Venice, at $48\frac{1}{8}$ d. per ducat? Answer 2880 ducats.

3. Venice owes to London 9579 ducats, 18 ſols, 6 deniers currency: For how much Sterling may London draw, the agio being 20 per cent. and the exchange 4s. 1d. per ducat banco?

Answer, L. 1629 18 $4\frac{1}{4}$.

Examples with Genoa.

1. Genoa is indebted to London L. 252 10 $1\frac{1}{2}$ Sterling: For how many pezzoes may London value upon Genoa, at $50\frac{1}{4}$ d. per pezzo.

Answer, 1206.

2. London imports to Genoa a parcel of goods, which are ſold for 4235 lires, 12 ſoldi, 5 denari: For how much Sterling may London draw, exchange $51\frac{1}{8}$ d. per pezzo? Answer L. 180 10 6.

Divide the lire-money by 5, gives the exchange-money.

Examples

Examples with Leghorn.

1. What is the Sterling value of 3655 piastres of Leghorn, $55\frac{1}{2}$ d. per piastre? Answer, L. 845 4 $4\frac{1}{2}$.

2. Leghorn is indebted to London 3807 piastres lire money: For how much Sterling will London draw, at 50d. per piastre? Answer L. 132 3 9.

Take a sixth part of the lire-money for the exchange-money.

3. How many piastres of Leghorn are equal to 1000 crowns of Naples, when 100 of the former are 134 of the latter? Answer $746\frac{1}{8}$.

4. How many crowns of Sicily, $133\frac{1}{3}$ soldi per crown, are equal to 1600 piastres of Leghorn?

Answer 240.

SIMPLE ARBITRATION OF EXCHANGE.

1. Suppose the exchange betwixt London and Dublin to be L. 112.2 Irish for L. 100 Sterling; and suppose the exchange with Glasgow at the same time be 2 per cent. in favours of London; what must be the course of exchange betwixt Glasgow and Dublin, to be at a par with the two courses given?

Answer 10 per cent.

$$102 : 112.2 :: 100 : 110$$

In drawing and remitting, observe always where the money is above par, or raised in value, and where it is lessened, and draw upon the latter place, and remit to the former.

2. If the course of exchange betwixt Britain and France be 32d. per crown; and betwixt Britain and Portugal 64d. per milree; and advice is received that exchange betwixt Portugal and France is 460 rees per crown:

crown: How must I draw and remit to these two nations, and how much, so as to gain L.100?

Answ. Remit to Portugal L.1150, draw on France for L.1250.

64 : 1000 :: 32 : 500 rees, the arbitrated par: so the money of Portugal is above par. Therefore draw upon France for 1 crown, you have 32d. for your draught; let France value on you to Portugal for that crown, debits you there 460 rees, that is, 29.44d. Sterling; which you remit to Portugal: so you have gained by drawing, for 1 crown, 2.56d. Then say, 2.56 : 32 :: 100 : 1250; or thus: Remit to Portugal 64d. credits you in 1000 rees; let France value on you to Portugal, credits you in France 2.1739 crowns; draw on France for this, you have 69.564d. for your draught, 5.56d. profit. Then say, 5.56 : 64 :: 100 : 1150.

To prove this, say, .13 : 1 :: 1250 : 9375 crowns, debit at France; and 1 : 460 :: 9375 : 4312.5 mil-rees, debit at Portugal; and, 1 : .26 :: 4312.5 : 1150 to be remitted to Portugal.

3. When exchange betwixt Britain and France was 32d. per crown, and with Portugal, at 64d. per mil-ree, I drew upon Portugal, and remitted to France, and gained 8 per cent.: What was the course betwixt Portugal and France? Answer 540 rees per crown.

Reduce L.108 to rees for a dividend, and L.100 to French crowns for a divisor, your answer is as above; or see next example.

4. When exchange was with France and Portugal, (as in the last Example) I drew upon France, and remitted to Portugal, and gained 8 per cent. What was now the exchange betwixt France and Portugal?

Answer 462 rees per crown.

Reduce L.108 to French crowns for a divisor, and L.100 to rees of Portugal for a dividend, quotes

Q

462 $\frac{2}{3}$

$462\frac{1}{2}$ rees: Or perhaps more intelligibly thus, find the par, viz. 500 rees, and say, by the Rule of Three Inverse, $100 : 500 :: 108$.

5. A merchant at London is creditor at Rome 500 crowns; for which he can draw at London 6s. per crown; or he can have them remitted to Venice, at 100 ducats banco each 70 crowns, and then can draw on Venice, at 4s. $4\frac{1}{2}$ d. per ducat; but will be obliged to allow the Venice correspondent $\frac{1}{2}$ per cent. commission: Which will be most advantageous for him, to draw for them directly, or have them remitted by Venice?

Answer, To have them by Venice the advantage is L. 5 9 $4\frac{1}{2}$ d.

6. A merchant at London is creditor at Amsterdam for 6000 guilders, and at the same time at Paris for 4125 crowns. He is offered 22d. per guilder for the whole, provided he will bring the French money to Amsterdam; or he is offered 32d. per crown, if he will bring the Flemish money to Paris: Which offer is the best, exchange between Paris and Amsterdam being 56d. Flemish per crown?

Answer, The last offer is the best by L. 42 0 $0\frac{1}{4}$.

Find the par thus: $22 : 40 :: 32 : 58\frac{2}{3}$ so as the course is in favours of Holland, draw on France.

7. Amsterdam is indebted to Britain in 3000 bank guilders, for which the British merchant can draw, at 22d. per guilder; but, to make trial for the better, he orders his factor at Paris to value on Amsterdam, at 55d. Flemish per crown; who accordingly brings the money to Paris, retains $\frac{1}{2}$ per cent. commission, and remits the neat proceeds to Britain, at 32d. Sterling per crown: What advantage had the merchant by the money taking this tour? Answer L. 14 9.

8. A merchant at Amsterdam gives orders to his factor at London to remit him 5000 guilders, at $21\frac{1}{2}$ d.

$21\frac{1}{4}$ d. per guilder, and to raise a fund for this by drawing upon France, at 30d. per crown; but ere the order arrives, the course of exchange was altered; bills for Amsterdam being at $22\frac{1}{4}$ d. per guilder, and those on France daily advancing; whereupon the factor resolves to wait till he can perform the order to answer his constituents proposal, and at the same time save himself $\frac{1}{4}$ per cent commission: What must he draw for his French crown to obtain this end?

Answer $32\frac{1}{4}$ d.

Say, $3400 : 30 :: 3627 : 32.0029 +$ These numbers are found by multiplying $21\frac{1}{4}$ by 100, and $22\frac{1}{4}$ by 100.75, and abridging.

For a check upon this, consider, That 5000 guilders, at the assigned price, amount to L. 442 14 2 Sterling; which shews, that he is to draw for $3541\frac{1}{3}$ crowns, at 30d. But 5000 guilders, at the advanced price, make L. 468 15; upon which sum he is to make $\frac{1}{4}$ per cent. for this is the sum he remits, and therefore must draw L. 472 5 $3\frac{1}{4}$; and this divided by the number of crowns, gives as above.

9. A merchant at Amsterdam orders his factor at London to remit him 720 guilders, at $22\frac{1}{4}$ d. per guilder, and for that to draw upon Spain, at 40d. per piaſtre; but before this order was received, bills for Amsterdam were at 23d. per guilder, and at the same time those on Spain sold at 41d. per piaſtre: Whether can the factor remit the guilders consistent with the order or not? Answer, He can.

For $22.5 : 23 :: 40 : 40.8$ If he draws $40\frac{8}{9}$ for his piaſtre, he can remit much more when he draws 41d. for he will have $\frac{1}{9}$ d. profit on every piaſtre, or 3s. 9d. on the whole.

10. A merchant at Paris orders his agent at London to remit him 1000 crowns, at $31\frac{1}{2}$ d. per crown, and to draw for the value upon Amsterdam, at 35s. Flemish

mish per pound Sterling; but when the order came up, bills on Paris were at $30\frac{1}{4}$ d. per crown: What must the exchange with Amsterdam fall to, to compensate the advance on the remittance; also, how much Sterling pays the remittance, and how much Flemish the draught? Answer, You may state thus:

$30.5 : 35 :: 30.75 : 34.71544$ This Inverse.
For if he has to pay more Sterling for his French money, he must also receive more Sterling for his Dutch money, otherwise his employer would be a loser.
L. 128 2 6 pays for the remittance, and L. 222 7 11 Flemish the draught.

COMPOUND ARBITRATION OF EXCHANGE.

1. If L. 1 Sterling be equal to 420d. Flemish, 54d. Flemish to 1 French crown, 100 French crowns to 54 rubbles of Denmark; how many rubbles of Denmark are equal to the pound Sterling? Answer $5\frac{1}{5}$.

It might be wrought at large, thus: $54 : 1 :: 420 : 7.7$ crowns, and $100 : 54 :: 7.7 : 5.2$ rubbles, the value of the pound Sterling.

Or rather set the antecedents and consequents in two columns, thus:

<i>Antecedents.</i>	<i>Consequents.</i>	<i>Abridged.</i>
1 L. Sterling	= 420 d. Flemish.	1 21
54 d. Flemish	= 1 French crown.	. 1
100 French crowns	= 54 rubbles.	5 .
How many rubbles	= 1 L. Sterling?	1

So 21 divided by 5 answers the question.

2. When exchange betwixt London and Amsterdam is 35s. Flemish per pound Sterling; betwixt Amsterdam and Paris 58d. Flemish per crown; from Paris to Venice 100 crowns per 60 ducats banco; from

from Venice to Madrid 360 mervadies per ducat:
How much Sterling is the value of the piaſtre of 272
mervadies? Answer $41\frac{1}{8}\frac{2}{9}$ d.

		<i>Abridged.</i>	
240d. Sterling	= 420d. Flemish.	4	7
58d. Flemish	= 1 crown.	58	1
100 crowns	= 60 ducats.	1	3
1 ducat	= 360 mervadies.	34	9
272 mervadies	= How many d. Sterl.?		

Then 4, 58, and 34 multiplied, make 7888, and 7,
3, and 9 make 189; by which divide, and you have
the par of the piaſtre, which is nearly $41\frac{1}{4}$. If it had
been required, How many piaſtres for a pound Ster-
ling? it would be found thus:

		<i>Abridged.</i>	
1 L. Sterling	= 420 d. Flemish.	29	21
58 d. Flemish	= 1 crown.		3
100 crowns	= 60 ducats.	17	45
1 ducat	= 360 mervadies.		
272 mervadies	= 1 piaſtre.		
How many piaſt.	= 1 L. Sterling?		

And multiplying and dividing, as before, you have
 $5\frac{3}{4}\frac{7}{9}\frac{2}{3}$ piaſtres, for the value of the pound Sterling.
Now, ſuppoſing the exchange between London and
Madrid to be 43d. per piaſtre, and betwixt all the
other places the ſame as in the above example; then
remit L. 1000, by way of Amſterdam, Paris, Venice,
and ſo to Madrid, as in the queſtion; ſtate as before;
putting 1000 in place of 1, and you will find it credit
you in Spain L. 1030 $5\frac{1}{4}$: ſo have you made L. 30,
&c. of profit. But, again, if the courſe with Spain
has fallen, for inſtance, to 40d. then alter your courſe,
and remit to Spain, from thence to Venice, Paris and
Amſterdam; laſtly, draw on Amſterdam: the opera-
tion in this caſe will ſtand thus:

40d.

40d. Sterling	=	272 mervadies.	<i>Abridged.</i> 68
360 mervadies	=	1 ducat.	9 25
60 ducats	=	100 crowns.	29
1 crown	=	58d. Flemish.	80
How many d. Flemish = 240000d. Sterling?			

These multiplied and divided by 9, gives 436222.2d. Flemish; for which, at 35s. the supposed course you will draw L. 1038 12 5 $\frac{1}{4}$ Sterling; so that the gain is L. 38 12 5 $\frac{1}{4}$.

3. A merchant at London is creditor at Madrid for 5000 piaſtres, for which he could draw directly, at 50d. per piaſtre; but hoping to make ſome more of them in the circular way, he orders them to be remitted to Venice, at 94 piaſtres for 100 ducats; and from thence to Cadiz, at 320 mervadies per ducat; thence to Liſbon, at 630 rees per piaſtre of 272 mervadies; thence to Amſterdam, at 50d. Flemish per cruſade of 400 rees; thence to Paris, at 56d. Flemish per crown; and, laſtly, draws upon Paris, at 32d. Sterling per crown: How much did he gain by the money taking this courſe? Answer L. 131 13 6.

For to have drawn directly on Madrid, he would have received L. 1041 13 4; then for the circular way ſtate as follows:

94 piaſtres	=	100 ducats.	<i>Abridged.</i> 47 25
1 ducat	=	320 mervadies.	17 10
272 mervadies	=	630 rees.	45
400 rees	=	50d. Flemish.	25
56d. Flemish	=	1 crown.	16
1 crown	=	32d. Sterling.	50
How many d. Sterl. = 5000 piaſtres?			

You get for answer L. 1173 6 10; from which ſubtract L. 1041 13 4.

4. A mer-

4. A merchant at London is creditor at Lisbon for 5000 milrees, for which he can draw directly 5s. 8d. per milree; or he can have them remitted to Madrid, at 630 rees per piastre, of 272 mervadies, and draw upon Spain, at 43d. per piastre; or he can have the money further remitted to Paris, at 182 mervadies per crown, and can draw upon Paris, at 32d. per crown; or from Paris can have them brought to Amsterdam, at 50d. Flemish per crown; and can draw on Amsterdam, at 38s. Flemish per pound Sterling: Whether shall he draw for his money directly on Lisbon, or bring it the course we have mentioned, or stop at some part of the round, so as to make the most gain by it? Answer,

In the first place, By drawing on Lisbon, he would have for his draught L. 1416 13 4; and if he has them remitted to Madrid, say, 630 : 1 :: 5000000 : 7936.507 piales, which, at 43d. make L. 1421 19 1 $\frac{1}{4}$: so by bringing them to Spain, he makes L. 5 5 9 $\frac{1}{4}$ advantage; for the arbitrated par here with Spain would be 42 $\frac{1}{3}$ d. so that he would gain $\frac{1}{3}$ d. on every piastre. Now, supposing him to remit the money to France, work thus:

<i>Abridged.</i>		
630 rees	= 272 merv.	63 136
182 mervadies	= 1 crown.	91 32
1 crown	= 32 d. Sterl.	
? d. Sterling	= 5000000 rees	500000

The amount here turns out L. 1581 9 8 $\frac{3}{4}$, which is L. 159 10 7 more profit than before. Now, let us suppose him to remit the money to Amsterdam:

<i>Abridged.</i>		
630 rees	= 272 mervad.	63 136
182 mervad.	= 1 crown.	91 5
1 crown	= 50 d. Flemish.	57 625000
456 d. Flem.	= 1 L. Sterling.	
? L. Sterl.	= 5000000 rees.	

Multiply

Multiply and divide, and you will find the answer to be L. 1300 11 $3\frac{1}{2}$; so that, by bringing them to Amsterdam, he would lose L. 280 18 $5\frac{1}{2}$ that he might have drawn for them at Paris: and when he perceives this, he will certainly stop short of bringing them there.

5. A merchant at London is creditor at Madrid for 1000 piaftres, for which he can draw 51 d. per piaftre; but, in order to advantage, he caufes them to be remitted to Venice, at 94 piaftres per 100 ducats; from thence to Paris, at 60 ducats per 100 crowns; thence to Amsterdam, at 56 d. Flemish per crown; and, laftly, draws on Amsterdam, at 36 s. Flemish per pound Sterling: What is the arbitrated price of the piaftre betwixt London and Madrid, and how much is the circular better than the direct remittance?

We fhall ftate and abridge in a different form:

$$\frac{100 \times 100 \times 56 \times 240}{94 \times 60 \times 1 \times 432} = \frac{50 \times 5 \times 7 \times 240}{47 \times 3 \times 54} = \frac{25 \times 5 \times 7 \times 80}{47 \times 27}$$

and 1269)700000($55\frac{195}{1269}$ d. Sterling is the price of the piaftre; and this multiplied by 1000 piaftres, makes L. 229 16 $9\frac{1}{2}$? whereas by the direct draught, he could only have had L. 212 10: fo there is a gain of L. 17 6 $9\frac{1}{2}$ by the circular remittance.

In all cafes of circular remittances, it will no doubt happen, that the correspondents at the feveral places muft be allowed for their trouble: in which cafe their commiffion muft be fubtracted all along from the feveral confequents. Thus, refuming the laft Example, and fupposing every agent to retain $\frac{1}{2}$ per cent. fubtract the 200th part from every confequent, as under.

94 piaftres	=	99.5 ducats.	<i>Abridged.</i>
60 ducats	=	99.5 crowns.	47 99.5
1 crown	=	55.72 d. Flem.	9 19.9
432 d. Flemish	=	240 d. Sterling.	. 2.33
? d. Sterling	=	1 piaftre.	. 5.

One piaftre this way will be found worth 54.533d. and fo 1000 will amount to L. 227 4 5, fomewhat lefs than before; but there is ftill a profit.

6. A merchant at Glasgow is creditor in Jamaica for the neat proceeds of a cargo of goods, for which he can draw L. 100 Sterling for 140 of currency; but orders his factor to remit the money to Boston, at L. 160 Jamaica currency for L. 180 Boston currency; and again by his order, the money is remitted to Holland, at L. 1 Boston currency for 25s. Flemish; and from Holland to France, at 60d. Flemish per crown; and, laftly, draws on France for the whole: I demand how much he muft draw for the crown, fo as to gain 5 per cent. by the circular remittance?

Answer, 32d.

The numbers might here be arranged in the fame order they come in the question; for the par of any intermediate term may be found by the rule: But yet I think the moft natural way is to ftate thus:

1 French crown	=	.25 L. Flem.	<i>Abridged.</i>
25 L. Flemish	=	20 L. at Boston.	3. .05
180 L. Boston	=	160 L. Jamaica.	. 2
100 L. Ster. in credit	=	105 L. to be drawn.	. 4
140 L. Jamaica	=	100 L. Sterling.	
What Sterling	=	1 French crown.	

And 3).40(.133 L. or 32d.

This may soon be proved at large, thus:

160 : 180 :: 140 : 157.5 at Boston.

20 : 25 :: 157.5 : 196.875 Flemish.

.25 : 1 :: 196.875 : 787.5 French crowns.

and 787.5 : 105 :: 1 : .133 or 32d. as before.

R

It may serve for another example, to suppose the factors at Boston, Holland, and Paris to retain each 1 per cent. commission, and then let it be required, What he must draw for his French crown, so as to gain 5 per cent. still? The answer will be found very near 33d. viz. 2s. 8d. 3.9qrs.—We leave the operation for the learner's exercise; and so conclude this tedious, though very useful article of Exchange.

CHAPTER XIV.

RULE OF FALSE.

Quest. 1. A merchant, who had a free, small stock, falls to trade, and in a year's time triples his stock; off this he takes L. 100 of expences; triples the remainder the second year, spends L. 100 also; and does the same the third year, and then finds his stock triple what it was at first; I demand his original stock.

Answer L. 54 3 4.

Suppose any sum: make trial of it according to the conditions of the question; what you differ from the truth at last, call your first error; make trial with another number the same way; if you either exceed, or fall short of the truth both times, your errors are similar, and then the rule is, Multiply the first position and second error, also the last position and first error. If your errors were similar take the difference; or if dissimilar, the sum of these products for a dividend; also the sum or difference of the errors for a divisor; the quotient is the answer.

2. A merchant setting up trade, receives the first year L. 50 in a legacy, but loses L. 200 by trade; the next year gains L. 200, but loses L. 60 by suretiship; the third year doubles what stock he has on hand, and pays L. 150 for 3 years expences of house-keeping, and then finds himself just as he began: What was his stock? Answer L. 170.

3. I have three silver cups, which have one cover of 6 ounces, common to them all. This, if laid on the first cup, makes it weigh equal to both the other two; if laid on the second, it weighs equal the first cup; and if laid on the third, it weighs equal the half of both first and second: Required the weight of each cup? Answer 21, 15, 12 oz.

4. A drover, who had laid out L. 870 upon oxen and cows; paying L. 7 for every ox, and L. 5 for every cow; sells half of his oxen, and a third of his cows, to a butcher for L. 396; having gained 10 per cent. I demand how many oxen and cows he bought?

Answer 60 oxen, 90 cows.

$$110 : 100 :: 396 : 360$$

So half the oxen, and third the cows cost him L. 360. Then suppose he bought 80 oxen, they cost him L. 560, so must he lay out L. 310 on cows, which would buy 62. Now, $\frac{1}{2}$ of 80, and $\frac{1}{3}$ of 62, at L. 7, and L. 5, would amount to L. 383.3; which should have been L. 360: Therefore the error is 23.3 too much. Suppose 70 oxen, and the error will be 11.6. Then work by the rule.

5. Two men thus discourse of their money: Says A to B, Give me L. 20 of your money, and I shall have a pound for every shilling you have left; says B to A, Give me L. 12 of your money, and I shall have a penny for every shilling you have left: How much had each? Answer, A, L. 1020, B, L. 72.

6. A

6. A certain man being demanded his age, with that of his wife, son and daughter's; answered, My son's age is double that of my daughter, and my own age is double them both; but if we are spared together 40 years longer, my age will be only equal to them both: What was the age of each?

Answer 80, $26\frac{2}{3}$, $13\frac{1}{3}$ years.

7. There is in a large room, a company of men and women with several tables. If the men and half the women sit down, they make 8 at each table; if the women with half the men sit, they make 7 at each table; but calling for 4 tables more, they sit all down, and make 6 at each table: I demand the number of men, women and tables?

Answer 36 men, 24 women, 6 tables.

It appears here the men with half the women, are in proportion to the women and half the men, as 8 to 7; and these two sums added, must be $1\frac{1}{2}$ times the number in company; if we call them 7 and 8, they make 15; from which take $\frac{1}{3}$, the company will be 10; and $10 - 8 = 2$ half the women, and $10 - 7 = 3$ half the men: So the men are to the women as 3 to 2. Now, suppose any convenient number in company, as 20, then must the men be 12, the women 8, and one of these, with half the other, make 16, or 14; the first divided by 8, or the other by 7, gives 2 tables; to which add 4, makes 6, and 6 times 6 is 36 in the company; but you supposed 20, so your first error is 16, &c.

8. One being asked, What of the clock? answered, The number of the month, hour and day make up 40, and their continued product is 1500; also the sum of the day and hour make 7 times the number of the month, and the hour is the greatest number: What was the hour of the day?

Answer 20; that is 8 evening.

Suppose the month, and you will find it to be the 5th.—Then see Problem 6th of Extraction of Roots.

9. Sold 60 lb. of cinnamon, and 40 lb. of pepper, for L. 50 13 4; and again sold 70 lb. of cinnamon, and 50 lb. of pepper, for L. 59 6 8: I demand the value of a pound of each?

Answer, Cinnamon 16s. pepper 16d.

10. I have a pair of gold buckles, which weigh 5 oz. Troy; a pair of silver, cast in the same mould, that weigh 3 oz. I would mix as much of gold and silver, as would make a pair of the same largeness, and that might weigh 4 oz. How much of each metal must I mix? Answer $2\frac{1}{2}$ gold, $1\frac{1}{2}$ silver.

11. There is a cistern with 4 cocks, containing 840 gallons of water: The first cock could empty the cistern in 28 minutes; the second in 42; the third in 70; and the fourth in 105 minutes: Required in what time the cistern would be emptied, all four running at once? Answer, In 12 minutes.

12. There is a heap of money, consisting of 960 pieces; halfpennies, shillings and six-pences: The halfpennies are half the number, but are only $\frac{1}{8}$ of the value: I demand the number of each piece?

Answer 120s. 360 six-pences, 480 halfpennies.

These two do with one supposition; this last a trifle.

13. A man lays out L. 1000 upon 110 packs cotton and flax; paying 40s. more per pack for his cotton than his flax; also he gains on the cotton 12, and on the flax 7 per cent. and so has gained L. 100 in all: How many packs of each did he buy?

Answer 60 of cotton, 50 of flax.

Suppose he laid out L. 500 on each; his gain would be L. 95; so the error here would be 5, &c. And for the price of the packs (if you want to know it) suppose a pack of cotton L. 12, a pack of flax L. 10, would make 90 packs; so the error here would be 20, &c.

14. Two farmers unite in reaping their harvest: A has 2 reapers more than B; and his reapers are with

with B as many days as is the number of B's reapers: again, B's reapers are with A, a number of days equal to A's reapers; and this completes both their harvests: Now, supposing A to have 9 acres more than B, and that 4 men reap an acre per day, how many acres and reapers had each?

Answer, A 45, B 36 acres; A 10, B 8 reapers.

Suppose A had 6, and B 4 reapers, in 4 days they will reap 10 acres to B, so A must have 19; and with A in 6 day they will reap 15 acres, so the error is 4, &c.

15. One being asked, How many pounds of tea he had on hand? answered, 3 times as many as he had yet sold. Being asked, How many he had sold? he said, $\frac{1}{3}$ of those sold multiplied by $\frac{1}{2}$ of these remaining, would give all he had at first: Required the quantity at first? Answer 576 lb.

16. A company of travellers hire a number of carriages: Putting 3 men into one, and 4 women into another; they needed 32; but, at next stage, they put 4 men into one, and 3 women into another, and now needed but 31 carriages: How many men and women were there? Answer 60 men, 48 women.

17. A says to B, When we were first acquainted, I was triple your age, and at that time the sum of our ages was below 100 as far as it is above it now, when I am only double your age; What are their ages?

Answer, 80 and 40.

18. A had L. 100, B, L. 48: a band of robbers took from A thrice as much as they took from B, and yet left him twice as much as they left B: What took they from each? Answer, From A L. 12, from B 4.

19. With $\frac{2}{3}$ of my money I bought 20 bales of goods; and again 20 more, 5 per cent. dearer, and had L. 15 of my $\frac{2}{3}$ left, taking the whole of my money,

ney, I bought 20 more, 10 per cent. higher than at first, and then had L. 5 left: What was my money?

Answer L. 740.

Make supposition of the price of a bale, 63 times that must be your money, all to L. 5, and 41 times that with L. 15, must be $\frac{1}{3}$ of it; or if not, this is your error.

20. A man, who had 100 pieces, viz. guineas and shillings, said, If he had guineas for shillings, and shillings for guineas, his money would be quadrupled: Required the number of each?

Answer, 17 guineas, 83 shillings.

21. What two numbers are those whose sum is double, and whose product is 12 times their difference?

Answer 8 and 24.

Take 2 numbers, one double the other; if you make the least your position, then 12 times their difference divided by the greater, will either be the least, or the difference is your error.

22. What two numbers are these, that, if you multiply the one by 3, and the other by 4, the sum of the products will be 100; but, if you shift the factors, the difference of the products will be 100?

Answer 4 and 28.

23. My father is 20 years older than myself, and I am 20 years older than my son; again, my mother is 5 years younger than my father, my wife 5 years younger than myself, and my daughter 5 years younger than my son; the sum of all our years is 200: Required the age of each?

Answer, The father 55 years, 10 months, &c.

Many questions, that may belong to this Rule, may be easier wrought without it, such as these:

24. What number is that, to which, if you add 11, multiply the sum by 11, and from the product subtract

subtract the given number, the remainder will be 1111? Answer 99.

For 11 being added, and the sum multiplied by 11, the product must consist of the square of 11, together with the product of the required number by 11, and as the said number is subtracted, the remainder will be 10 times the number above the said square: so from 1111 take 121, divide the remainder by 10.

25. Two men have about them unequal sums of money: If the first man's were doubled, their sum would be L. 214 8 10; but if the second man's sum was doubled, the sum of both would be L. 237 2 8: What sum had each?

Ans. The first L. 63 18 4, the second L. 86 12 2.

Subtract the one given sum from the other, take this difference from the lesser sum, or add it to the greater, and the remainder or sum, divided by 3, gives the sums required.

26. Sent my whole stock to sea, save L. 100, which I paid as premium of insurance, and received $\frac{1}{4}$ increase from abroad; I did thus a second, and third time, and then found my stock doubled: Required my stock at first? Answer L. 1480.

If you work this by positions, it is easier to trace the working backwards; that is, suppose double the stock, subtract $\frac{1}{4}$, and add the L. 100, &c.

Any question of the like kind will work by this proportion, $\frac{15}{74} : 120 :: \frac{1}{4} :$

Note, The 120 is the L. 100 paid every voyage.

27. A privateer having gone out a-cruizing, took a prize of French crowns: Half of which being divided among the crew, each had 84; but afterwards 5 of them dying, there was a new division, and each had
now

now 94: I demand the number, both of the crew and crowns?

Answer, The crew 47, the crowns 7896.

Here 5 shares make 10 crowns to each of the survivors: Therefore $84 \times 5 = 420 \div 10 = 42$ the survivors.

28. A company at a tavern, upon numbering heads, find their reckoning to be 15s. a-piece; but, going about to lift the money, two of them happened to have none, which occasioned the rest to pay 1s. a-piece more: I demand the number in company?

Answer 32.

29. A man has a quantity of strong spirits, into which having put 40 pints of water, he can sell a pint for 3s. but if he puts in 50 more, he can sell a pint for 2s. I demand the quantity and price of the spirits?

Answer 60 pints, at 5s.

With a little consideration of the prices, you will find the last quantity of water must be $\frac{1}{3}$ of the whole.

30. Bartered wheat for barley, and gave 4 quarters for 5; and afterwards, wheat rising 5s. per quarter, and barley 2s. 6d. bartered again, and gave 3 for 4: I demand the rate of each grain?

Answ. Wheat 25s. barley 20s. per quarter.

Say, $4 : 5 :: 3 : 3.75$ what he would have got at the first rate. Whereas he got 4, viz. $\frac{1}{4}$ more, the 5s. the advance on the wheat, multiplied by the number of quarters, is 15s. and 2s. 6d. by the bolls of barley, is 10s. the difference 5s. is the price of $\frac{1}{4}$ of the quarter.

Most questions I find in other books, may be answered by such methods. I shall mention a few.

31. The money staked by A, B, and C playing at Hazard, was 324 crowns; but happening to disagree,
S each

each seized as many of the crowns as he could: A got a number unknown, B as many, and 15 more, and C got a fifth part of both their sums: How many crowns got each?

Answer, C got a sixth part, as the question plainly shews; and take 15 from the rest, A will get a half, &c.

32. A storemaster, who had 2000 lambs, paid the tithe: Lost at one time $\frac{1}{5}$ so many as he has now left; again, $\frac{1}{3}$, and again, $\frac{1}{4}$ so many: How many has he left? Answer 864.

Deduce the tithe, then $1800 \div 2\frac{1}{5}$.

The following questions are very silly.

33. A, B, and C built a house, which cost L. 76; of which A paid a certain sum, B as much as A, and L. 10 more, and C as much as A and B: How much paid each?

This question answers itself; for if C paid as much as A and B, he paid half of the whole.

And one still more trifling is this:

34. A certain man being asked the age of each of his 4 sons? said, There were 4 years betwixt the ages of each of them, and that the youngest was half the age of the eldest.

Here we are told, that one is double the age of another, and at the same time that the other is 12 years younger.

C H A P. XV.

EXTRACTION OF ROOTS.

TO extract the square root of any number, proceed thus: Put a point over every second figure, beginning at the units, if a whole number; or at the seconds, if a decimal, which divides your number into periods, seek the nearest root of the left hand period, and write it in the quotient; subscribe and deduce the square of that figure from the said period; and to the remainder bring down the next period for a resolvend; double your quote, and set it for a divisor to this; and now the same figure you put in the quotient, set also on the right of the divisor, and multiply the divisor so increased, by the quotient figure, and subtract its product; bring down another period, and go on the same way to the end; if there is a remainder, carry on the work by annexing ciphers, two at a time, as long as you please, for the decimal part of the root.—See an example, both of whole numbers and decimals.

$$\begin{array}{r} 54756 \text{ (234 root.} \\ \underline{4} \end{array}$$

$$\begin{array}{r} 43 \overline{)147} \\ \underline{129} \end{array}$$

$$\begin{array}{r} 464 \overline{)1856} \\ \underline{1856} \end{array}$$

(0)

$$\begin{array}{r} .3333 \text{ \&c. (.577} \\ \underline{25} \end{array}$$

$$\begin{array}{r} 107 \overline{)833} \\ \underline{749} \end{array}$$

$$\begin{array}{r} 1147 \overline{)8433} \\ \underline{8029} \end{array}$$

.404 &c

Square.

<i>Square.</i>	<i>Root.</i>	<i>Square.</i>	<i>Root.</i>
6091024	(2468	133096550976	(364824
1034008336	(32156	7804502671716	(2793654
25467	(159.5838 &c.	678423	(823.6643 &c.

§ 2. EXTRACTION OF THE CUBE ROOT.

There are different ways of procedure in extracting the cube root. The following I think as easy to remember as any.

Divide your number into periods, pointing first at the units; then miss two, and point one towards the left; or if it be a decimal, begin at the place of thirds, and point towards the right. Find the cube root of the left hand period, subtract its cube, and to the remainder bring down another period, for a resolvend. The square of the quotient, multiplied by 300, is your divisor. Ask how oft this divisor is to be found in the resolvend, and putting the figure in the quotient, multiply the divisor, and set the product under the resolvend; square the last figure of your quotient, multiply this by the other figures before it, and that again by 30, and set this under the former product, last set in the same place the cube of the last quotient figure, then add these three lines together, subtract their sum from the resolvend, and to the remainder bring down another period, for a new resolvend, which you are to manage just as before; when there is a remainder, annex periods of ciphers, to obtain the fractional part of the root. The most tedious part of the work is to square the quotient, when there are many figures in it for a divisor.—*For which see an easy method in the Algebraical part.*

Examples.

Examples.

<i>Cube.</i>	<i>Root.</i>	<i>Cube.</i>	<i>Root.</i>
194104539(579 125		142857,142 &c.(52275 &c. 125	
7500)69104		7500)17857	
52500		15000	
7350		600	
343		8	
60193		15608	
974700)8911539		811200)2249142	
8772300		1622400	
138510		6240	
729		8	
8911539		1628648	
(0)		620494 &c.	

<i>Cube.</i>	<i>Root.</i>	<i>Cube.</i>	<i>Root.</i>
12895213625(2345		312908547069(6789.	

The greatest use of the Extraction of Roots is in Mensuration. In the meantime, we shall set down some problems, drawn from Algebraical principles; which to those who have not learned that branch, will be some entertainment,

Prob. 1. *To find a mean proportional betwixt two numbers.*

Multiply the two numbers together, and extract the square root of the product.

Prob. 2.

Prob. 2. *To find two mean proportionals betwixt two numbers.*

Multiply the square of the lesser number by the greater, the cube root of the product is the first of two mean proportionals; and dividing this by the least extreme, you have the ratio; multiply the first found by the ratio, you have the second.

Prob. 3. *To find the first of three mean proportionals.*

Multiply the cube of the less extreme by the greater, the biquadrate root of the product is what you want.

Prob. 4. *To divide by extreme and mean proportion, any number.*

A number is said to be divided into extreme and mean ratio, when the lesser part has the same proportion to the greater, that the greater has to the whole.

The rule is, Square the number, add $\frac{1}{4}$ of itself, extract the square root, and from that subtract half the given number, the remainder is the greater part. No number will divide this way without a remainder; if you divide 17. 50, you will find 6.5 and 10.5, which is pretty near the truth.

Prob. 5. *The sum of two numbers, and difference of their squares given to find the numbers.*

To the square of their sum add the difference of their squares; this divided by double their sum, quotes the greater number: Otherwise, the difference of their squares divided by their sum, quotes the difference of the numbers.

Prob. 6. *The sum and product of two numbers given, to find the numbers.*

From

From the square of half their sum subtract their product; the square root of the remainder, added or subtracted, to or from half their sum, gives the greater or lesser of the numbers.

Prob. 7. The product of two numbers, with the sum of their squares being given, to find the numbers.

Square half the sum of their squares; from this subtract the square of their product; the square root of the remainder, added or subtracted, to or from half the sum of their squares, gives the square of the greater or lesser number,

Prob. 8. The ratio of two numbers, and sum of their squares given, to find the numbers.

Divide the sum of their squares by the square of the ratio plus 1. the quotient is the square of the lesser quantity; for always the square of the ratio of roots will be the ratio of the squares.

Prob. 9. The product and difference of two numbers given, to find the numbers.

To their product add the square of half their difference; extract the square root; to which add, or subtract from it, the said half difference.

Prob. 10. The product and ratio of two numbers given, to find the numbers.

Divide their product by the ratio, the square root of the quotient is the lesser number; or multiply by the ratio, the square root of the product will be the greater.

Prob. 11. Having the ratio and difference of two numbers, to find the numbers.

Divide their difference by the ratio wanting one.

Prob. 12.

Prob. 12. *The sum of two numbers, and sum of their squares given to find the numbers.*

From the square of their sum, take the sum of their squares; take half the remainder from the square of half their sum; the square root of this last remainder, added or subtracted, to or from the said half sum, gives the greatest or least of the two numbers.

Prob. 13. *The sum of two numbers, with the product of their squares being given, to find the numbers.*

The square root of the product of the squares is the product of the numbers.—Then see prob. 6.

Prob. 14. *The sum of two numbers, and sum of their cubes being given to find the numbers.*

From the cube of their sum subtract the sum of their cubes; divide the remainder by triple their sum; subtract the quotient from the square of half their sum, and the square root of the remainder added to the same half sum, or taken from it, gives the numbers.

Prob. 15. *Four numbers are in Geometrical Progression, given, their product, and the product of the two least numbers, required the numbers.*

Divide their product by the square of the product of the two least numbers; the biquadrate root of the quotient is the ratio of the progression.—Then see prob. 10.

Prob. 16. My ploughs, oxen, and acres are geometrical proportionals; and if multiplied continually, their product is 46656, and their sum, if added, 258: What is the number of each?

Answer, 6 ploughs, 36 oxen, 216 acres.

Extract

Extract the cube root of their product, you have the number of oxen; for the product of 3 proportionals is the cube of the middle one. Then work by prob. 6th.

Prob. 17. Two men depart from one place; the one travels east 40 miles a-day, the other south 30 miles a-day: How far are they asunder in 6 days?

Answer 300 miles; viz. the square root of the sum of the squares of both distances.

Prob. 18. A general drawing up his men in a square, has 500 over; but increasing each rank by 1, he wants 71 to complete the square: How many men had he? Answer 81225.

Half the difference betwixt two squares that are next one another, throwing away 1, is the square root of the lesser.

Prob. 19. I would plant 80000 trees in a long square, the length to be double the breadth: How many trees in a row? Answer 200 and 400.

Say, $1 : 2 :: 80000 : 160000$ whose square root is the length.

Prob. 20. The height of a tree is 63 feet; it happens, that being broke by the wind, the top falls 21 feet from the root: How many feet of it stood, and how many fell? Answer 28 stood, 35 fell.

Subtract the square of 21 from the square of 63, divide the remainder by twice 63, the quotient is the standing feet.

Prob. 21. Given the height and distance of two towers, to find where a ladder must stand, to reach the top of either.

Subtract the square of one tower's height from the other; take the remainder from the square of the
T
distance;

distance; divide this last remainder by double the distance; the quotient is the distance from the higher tower. To find the length of the ladder, see prob. 17th.

Prob. 22. After dealing a hand of cards to each in company, there remained 25 in the pack; and dealing one more to each, there remained four hands in the pack: How many were in company, and how many cards had each?

Answer, 9 in company, 4 cards each.

$$52 - 25 = 27 + 4 = 31$$

$$\text{and } 52 - 31 = 21 + 2 = 10.5 \times 10.5 = 110.25$$

$$27 \times 4 = 108 \text{ and } 110.25 - 108 = 2.25$$

$$\text{and } \sqrt{2.25} = 1.5 \text{ and } 10.5 - 1.5 = 9 \text{ the hands.}$$

C H A P. XVI.

P R O G R E S S I O N.

NUMBERS are in Arithmetical Progression, which increase by the continual addition of some number, called the *excess*. Thus 3, 6, 9, 12, are progressions, where the common excess is 3.

Five things are to be considered here, viz. the *first term*, the *greatest term*, the *common excess*, the *number of terms*, and the *sum total of all the terms*; any three of which being given, the other two may be found, as in the following Propositions.

Prop. 1.

Prop. 1. In any arithmetical progression, the sum of the first and last, is equal to the sum of the two mean, or middle terms, when the number of terms is even.

Prop. 2. In any odd number of terms, the sum of the extremes is equal to double the mean, or middle term.

Prop. 3. If you subtract the least term from the greatest, and divide the remainder by the number of terms wanting one, the quotient is the excess.

Prop. 4. If you multiply the common excess by the number of terms wanting 1, and to the product add the least term, the sum is the greatest term.

Prop. 5. If you subtract the least term from the greatest, and divide the remainder by the excess, the quotient is the number of terms wanting 1.

Prop. 6. If to the last term you add what the first term wants of the common excess, or subtract what it exceeds it, and divide the sum, or remainder, by the excess, the quotient will be the number of terms.

Prop. 7. If you add the greatest and least terms into one sum, and multiply it by the number of terms, half the product will be the sum total of all the terms.

Prop. 8. If the total amount be divided by the half sum of the greatest and least terms, or if double the total be divided by the whole of that sum, the quotient will be the number of terms.

Prop. 9. If double the total be divided by the number of terms, the quotient will be the sum of the greatest and least terms.

Prop. 10. If the number of terms be even, subtract the product of the excess, and half the number of terms from

from the last term, double the remainder, and add the common excess, and you have the sum of the greatest and least terms; or if the terms are odd, throw away one, multiply half the remainder by the excess, subtract the product from the greatest term, the remainder is the middle term, which doubled, is the sum of the greatest and least.—*See prop. 2.*

Prop. 11. If from the sum of the greatest and least terms you subtract the product of the excess by the number of terms wanting 1, half the remainder will be the first term.

We now give an example to each of ten cases.

Case. 1. When the first term, the common excess, and number of terms are given, to find the last term and total.—*See prop. 4th and 7th.*

Example. A carter being asked what he would have to carry mile-stones to a toll-road 300 miles long, demands L. 100: I demand how many miles he must travel, supposing he carries but one at a time, how long time it will take him, driving 40 miles a-day, and that the stones are one mile from the end of the road, and what are his wages a-day?

Answer, 90300 miles, which is all we find by this rule. He will be $2257\frac{1}{2}$ days in performing the jobb, and his wages little above a penny a-day.

Case 2. When the first term, common excess, and last term are given to find the number of terms and total.—*See prop. 6th and 7th.*

Exam. When my wife and I were married, the first month after marriage, she spun 30 cuts of yarn, and has increased her thrift 20 cuts each month ever since, till this last month she made 490 cuts: I demand how long

long we have been married, and what is the whole quantity of yarn spun, and what her wages amount to, at 6d. a-dozen?

Ans^w. Married 24 months, the yarn 130 spindles, wages L. 13.

Case 3. *When first, and last terms, and number of terms are given, to find the excess and total.*—See prop. 7th and 3d.

Exam. I served with a master 16 years, putting the wages in his will: The first year he allowed me only 25s. which I thinking little enough, he promised to advance me so much more every year, till I was satisfied; he did so, and the last year of my service, I had guineas for shillings that I had the first: I demand what was the yearly advance of my wages, what the whole amount was, and how much it was every year at a medium?

Answer, The yearly advance 33s. 4d. amount L. 220, that is, L. 13 15 at a medium.

Case 4. *When first term, number of terms, and total are given to find the last term, and excess.*—See prop. 9th and 3d.

Exam. A footman is to travel from Edinburgh to London in 6 days, which is 402 miles the east road, travels the first day only 32 miles; but finding at this rate he will fall short of accomplishing his journey in time, he would know how much he must increase his journey each day, so as to answer his purpose, also what his longest day's journey will be?

Answer, Must increase 14 miles every day, his last day's journey will be 102 miles.

Case 5. *When first and last terms, and total are given, excess and number of terms required.*—See prop. 8th and 3d.

Exam. A

Exam. A and B are to pay betwixt them a debt of L. 10000 in a certain number of days; A undertakes to pay a certain sum every day, but B increases in arithmetical progression: Now, they pay betwixt them L. 77 10 the first day, and L. 322 10 the last; and then B has paid L. 2750 more than A: I demand how long they were in paying the whole, and how much each paid in a day?

Answer, 50 days, A paid L. 72 10 a-day, B, L. 5 the first, L. 10 the next, &c.

Case 6. When common excess, number of terms, and last term are given, to find the first term and total.—See prop. 10th and 7th.

Exam. A company at a tavern called a reckoning, and upon information of what bottles they had in, ordered the drawer to bring as many, and as the company was increased, 2 bottles more; they did so again and again, till at the 10th and last time, he brought them 5 dozen all at once: I demand how many bottles were at the first reckoning, and how many in all?

Answer, 42 at first, 510 in all.

Case 7. When number of terms, greatest term, and total are given, to find the least term, and the excess.—See prop. 9th and 3d.

Exam. An hundred trees that grew in equal distances in a straight line, being cut down, a carter has to lead them one by one to a place at some distance; well, he drives 20 miles a-day, is 51 days in performing the jobb, and the last tree takes him a whole day: I demand to what distance he carried them from the first tree, and how far it was betwixt the trees?

Answer 64 perches answers both.

Case 8.

Case 8. *When the excess, number of terms, and total are given, to find the greatest and least terms.*—See prop. 9th and 11th.

Exam. Sixty gentlemen in company fell to comparing their purses; they found that amongst them all they had L. 1000, and that the heaviest purse exceeded the next to it 11s. and so downward: I demand the greatest and least sums of money amongst them?

Ans. The greatest L. 32 17 10, the least 8s. 10d.

The two following cases require particular rules.

Case 9. *When first term, common excess, and total are given, and last term and number of terms required.*

Multiply the total by double the excess, and to the product add the square of half the excess; multiply the first term by the difference betwixt it and the excess; add this product to the former sum, if the first term was greater, but subtract it from it, if it was less than the excess; extract the square root of that sum, or remainder, and from the root take half the common excess, the remainder is the greater term.—Then see prop. 8th.

Exam. How long would a stone take to fall from the top of the atmosphere to the centre of the earth, calling it 4500 miles, and supposing the stone to descend 5 miles the first minute, and to increase its velocity 10 miles every minute? Answer, 30 minutes.

Case 10. *The greatest term, excess, and total given, to find the least term, and number of terms.*

Add the excess to the greater term, multiply the sum by the greater term, call this your first product; multiply the total by double the excess, call this your second product; if the last product be greatest, subtract
the

the first from it, subtract the remainder from the square of half the excess, and the square root of this last remainder taken from half the excess, leaves the first term; but if your first product was greatest, subtract the last from it, to the remainder add the square of half the excess, the square root of this sum added to half the excess, is the first or least term of the progression.—Then see prop. 8th.

Exam. Off 20 acres of land, in good heart, I reaped in one year 200 bolls of grain; but upon repeating the crops, I found it to inlack 10 bolls every year, and now I know that in all I have reaped 2000 bolls: Can you tell me how many years I have kept it in tillage?

Answer 16 years.

Some promiscuous Examples follow:

1. How many strokes does the clock strike in a year's time. Answer 56940.

2. There is a set of privileged poor in Scotland, called *Blue-Gowns*; every one of which, on the king's birth-day, at Edinburgh, amongst other emoluments, receive a penny for each year of the king's age: I demand how much one of them received during the reign of George the Second, who was 44 years old when he began to reign, and reigned 33 years?

Answer, L. 8 5.

3. Two ships sail from Greenock to Boston; the one sails 7 miles a-day more than the other, and arrives 301 miles before her: I demand the rates of sailing, calling the distance 2150 miles?

Answer, The first ship sails 50 miles a-day, the other 43.

4. A ship setting out to sail round the world on the line, sails the first day 17.28125 miles, that is, 17 miles, 2 furlongs, 10 perches, and increases her rate of sailing every day 20 perches: In how many days will she accomplish the voyage. Answer, In 600 days.

5. A woman

5. A woman, who, from the time of her marriage, had brought a child every 2 years, said, When her seventh child was born, that all her childrens ages were equal to her own: What was her age?

Answer, 42 years.

6. Two persons that are north and south from one another, set out to meet each other, the one travels the first day 60 miles, the other 48; but through fatigue each of them shorten their journey 4 miles every day, till they give over altogether, and then they are still 208 miles asunder: Required their distance at first?

Answer, 1000 miles.

7. Divide L. 20 among 20 men, so as each one following may have 20d. more than the foregoing?

Answer, Give the first 50d. the next 70d. &c.

8. Divide two hundred pounds for me,

'Mong five and twenty men,

Let every difference equal be,

The greatest sum be ten?

Answ. Give the first L. 6, the second L. 6 3 4, &c.

9. There is a long row of corn-sheaves, each 6 yards from another; my neighbour and myself have to carry them one by one to a barn, 20 yards distant from the first; we do so; I bring the first sheaf, and when we have done, find he has travelled 600 yards more than I: Required the number of sheaves?

Answer, 100 sheaves.

10. Thirty-two men fall to work, to cut down 500 acres of grass, each can cut an acre in a day, but one of them gives over every day till it is finished: I demand in what time this will be, and how many will be at the last day's work?

Answer 25 days, 8 work the last day.

11. A piece of ribband is wrapped round a stick of 1 inch diameter, 60 times, and then the diameter of the roll is 3 inches: Required the length of the ribband? Answer 373.815 inches nearly.

U

12. There

12. There is a round field, an English mile in circumference, how oft must a plough go round it to plow an English acre, with a furrow 6 inches broad?

Answer 16.57 times nearly.

13. Two persons set out at one time; the one goes 7 miles a-day, the other 1 mile the first day, 2 the next, 3 the next, &c. When will he come up with the first? Answer, In 13 days.

For the first and last days travel must be 14.—
See prop. 2, 5, &c.

14. Two persons set out at one time; one goes 24 miles every day, the other 5 miles the first day, 10 the next, 15 the next, &c. When will he come up with the first? Answer, In $8\frac{1}{2}$ days.

Upon trial you will soon find he takes more than 8, and less than 9, and making these your supposed numbers, by way of the Rule of False, you will find as above; and for proof, you will find each to have travelled $205\frac{1}{2}$ miles.

15. A stone being dropt in water, descends 1 foot the first second, 3 the next, 5 the next, &c. after half a minute a bullet of lead is dropt, which descends 2 feet the first second, and increases its velocity 4 feet every second, reaching the bottom at the same time with the stone: The depth of the water is required?

Answer $1049\frac{1}{2}$ feet.

The stone has descended 900 feet before the bullet is dropt, and gains after that 900 feet more, this makes 60 terms of a progression, or 60 seconds; after this the bullet gains 1, 3, 5, &c. and as it has 1800 to gain, it will require 42 seconds more. Mind last example. The stone takes 102 seconds to descend, the bullet $72\frac{1}{2}$.

16. A curling-stone being played off with vast force, runs the first second at the rate of a mile per minute, the

the next it abates of its rapidity the 20th part, the next the 19th part, and so downward till it give over: How many feet will it run? Answer 924 feet.

17. My farms, my houses, and my ploughs,
My horses, oxen, and my cows,
My mowers, reapers, and my rakers,
Hogs, scores of sheep, and labour'd acres,
My planted trees, my thraves unthreshen,
All in Ar'thmetical Progression;
Make up just six and fifty score
In one round sum but less or more;
Who is, and where's the man, can find
The number out of ev'ry kind?

This question is ambiguous, and admits of various answers, for only the number of terms and total being given; 80 will be found an arithmetical mean betwixt the middlemost terms: So if you suppose the seventh term 79, and the eighth 81, and continue the terms both upwards and downwards to 7, you have one answer. But the excess may be any number from 2 to 12.

II. IRREGULAR PROGRESSION.

A question commonly given in Arithmetical Progression is as follows: Let 100 eggs lie in a straight line, each a yard from another, and the first a yard from a basket; and if a man were to gather the eggs, one by one, into the basket, how many yards would he travel?

The answer to which is 10100 yards, and is a mere trifle; but I will propose the question in a way rather more difficult:

Let 100 eggs lie in a straight line, but at unequal distances, thus: The first 1 yard from a basket, the second 2 yards from the first, the third 3 yards from the

the second, &c. the last 100 yards from the ninety-ninth: How many yards must a man travel, to bring them, one by one, into the basket?

Answer 343400 yards, or 195 miles, 200 yards.

The terms of the progression will stand thus: 2, 6, 12, 20, 30, 42, 56, &c. where we may observe, that the excess always increases by 2, which may be called its variation; also, that any term here, is the sum total of the same number of terms in the common question; and an easy method to find any term, is to multiply the number of the term by itself, plus 1, and for the sum total, find the last term of an arithmetical progression, whose first term is 1, the excess $\frac{1}{3}$, and of the same number of terms as the given one, and multiply the last term of the one by that of the other; so here the terms being 100, multiply by 101, you have the last term; and the 100th term of an arithmetical progression, whose first term is 1, and common excess $\frac{1}{3}$, will be found, by prop. 4. last section, to be 34, by which multiply 10100, gives the answer. I found another very curious rule for finding the total; but it being somewhat tedious, I omit it.

2. Let 1000 shillings lie in a straight line, thus: The first 1 yard from my foot, the second 2 yards from the first, the third 3 yards from the second, &c. If I were to bring these shillings, one by one, to where I stand, and have them to myself, what would my wages be, one day with another, suppose I travel 40 miles a-day? Answer $2\frac{1}{2}$ d. +

If you work, as in the last example, you will find the number of yards to be 334334000—the rest is done by the Rule of Three.

If the number of terms were required here, and the last term given, extract the square root of the last term, and either the quotient or remainder will be the number of terms. If the sum total be given, multiply

multiply it by 3, extract the cube root of the product, neglecting the remainder. From the total found in this variety, it may also be had in these that follow, and many others that might be enumerated.

Variety 2. Suppose the first distance to be 1, (as before) the next 10, the next 20, the next 30, &c. the terms of the progression would stand thus, 2, 22, 62, 122, &c. where the variation of the increase is 20. To find the total of any number of these terms, take the total of one term fewer in the former variety, multiply it by 10, and add the product of the terms by the first term. To find any term, multiply ten times the number of terms by the number of terms wanting 1, and add the first term.

Var. 3. Let the terms be 2, 3, 5, 8, 12, where the variation is 1. To find the total, take half that found in the first variety, and subtract the number of terms wanting 3, multiplied by half the number of terms. To find a term, multiply the terms wanting 1, by half the terms, and add the first term.

Var. 4. Suppose the terms to be 2, 5, 9, 14, 20. Take half the total of the first variety, and add the product of the terms, plus 1, by half the terms; and to find any term, multiply its number plus 1. by half the number, and again add the number itself.

Var. 5. If the terms were 2, 7, 14, 23, 34, multiply the terms minus unity by half the terms, add this to the total in the first variety; and to find a term, square its number, also the next lesser number, and add the difference of their squares to the greater.

Var. 6. Let the terms be 4, 10, 18, 28. Multiply the number of terms by itself plus 1, and add the total of the first variety; and for a term, multiply their number by itself plus 3.

Var. 7.

Var. 7. If the terms were $2\frac{1}{3}$, $5\frac{1}{3}$, 9, $13\frac{1}{3}$, where the variation is $\frac{1}{3}$, multiply the number of terms plus 1, by 5, and this by half the terms, add this to the total in the first variety, and take a third of the sum; to find a term, multiply the number of the term by itself plus 6, and divide the product by 3; to find the number of terms, multiply the last term by 3, extract the square root, and abating 2, you have the number of terms.

Varieties are endless here: We therefore proceed to another Case of a far different nature; for this rule turns out a very complex one.

CASE II.

Exam. 1. Suppose a storemaster to have 100 ewe lambs, and each of these to bring a lamb, at 2 years old, and these again in the same manner: How many would they amount to in 20 years?

Answer, 1094600.

If you set down the terms by way of progression, you always add the two last terms together, and you need only work for one, and multiply the last term by 100. The terms stand as under:

1, 1, 2, 3, 5, 8, 13, 21, 34.

It would be no great trouble to continue them far enough this way. However, if you multiply any term by 11, and add that term which is 5 distant going before, you have the term that is 5 distant following it; or if you multiply any term by 123, and subtract the 10th foregoing, you have the 10th following: so here, if you multiply 8, the 6th term by 11, and add the 1st, you have 89, the 11th term; and multiplying this by 123, and subtracting the first, you have 10946, the 21st term. There must be 21 terms, though the
years

years are but 20; for you may observe, that our 3d term is but the increase at the second year's end; understand the like reason in some other subsequent cases.

But if we had supposed them all to die at 9 years old; or, after they had brought 8 lambs, we must have subtracted from the total, found above, those that died yearly, together with those that would have descended from them had they lived; this would make ten different progressions, the first of which would have 11 terms, the second 9, (for none die the tenth year) the third 8, and so downward; also the first term of each must be double those that die, for both they and their young are wanting that year; but the first increase is only those that die, and the sum of all these progressions, taken from the whole found at first, gives the number alive at the end of the time.

But there is no need for setting down all these progressions, but only the first; for the second is the same wanting two terms, the third the same wanting three, the fourth is the seventh term doubled, and so all may be found by multiplying the respective terms by those that die yearly, thus:

$$\begin{array}{rcl}
 2 \times 34 & = & 68 \\
 3 \times 21 & = & 63 \\
 5 \times 13 & = & 65 \\
 8 \times 8 & = & 64 \\
 13 \times 5 & = & 65 \\
 21 \times 3 & = & 63 \\
 34 \times 2 & = & 68 \\
 55 \times 1 & = & 55 \\
 89 \times 1 & = & 89 \\
 144 \times 0 & = & 0 \\
 233 \times 1 & = & 233
 \end{array}$$

All these added, make 833 for the decrease to be subtracted, but differs 2 from the truth; which is occasioned by our being come to the end of a second age; for the first that died at 9 years, could die but once: so there are but 33, instead of 34, to die at the 19th year. So from 10946, found at first, take 831, and multiply by 100, gives the true answer, 1011500.

Of these multiplications above, you see that three of the products are the same with other three; which therefore might have been spared; and that other two of them, 55 and 89, are the same as 144, the last term but one of the first progression: So that nearly one half of the work might have been spared; which would be a great contraction, if the terms were numerous.

The sum total of all the terms was not wanted here, but might be had thus: Find two terms of the progression more than sought; from the last of these take the first of all, and you have the total wanted.

Exam. 2. Sixty gentlemen pay towards a voluntary contribution, as under, viz. The first 1 piece of money, the second 1 also, the third 2, and the fourth 3, &c. every one as much as both the two that preceded him: What will the contribution amount to?

Answer 4052739537880 pieces.

Because the terms are numerous, we will use a greater multiplier; for if you multiply any term by 15127, and from the product subtract the 20th term foregoing, you have the 20th following. The terms here as before, 1, 1, 2, 3, 5, 8, 13, 21. Now, multiply the 12th term by 123, and abate 1, you have the 22d term; multiply this by 15127, and abate 1, you have the 42d term; and that multiplied by 15127, and the 22d subtracted, gives the 62d term; from which take 1, you have the sum total of 60 terms, as above.

above. If the 60th term were wanted, it might be found in the same manner to be 1548008755920.

CASE III.

Exam. How many will 7 ewe lambs amount to in 20 years, producing at 2 years old, and always 2 at a birth? Answer 4893357.

Here the terms proceed by adding the last to the double of the one before it, or perhaps easier, by doubling the last term, and either adding or subtracting the first, according as an odd or even number is wanted.

1, 1, 3, 5, 11, 21, 43, 85.

If you multiply any term by 32, and either add or subtract 11 times the first term, as an odd or even term is required, you have a term 5 distant. So, if you multiply 11, the fifth term, by 32, and subtract 11 from the product, you have the tenth term 341. Multiply any term by 1024, add 341 for an even, or subtract it for an odd term, you have one 10 distant: This is supposing the first term to be 1. So multiply the 10th term 341 by 1024, and subtract 341, you have the 20th term; and this doubled, adding 1, gives the 21st term; which, multiplied by 7, gives as above. If you make 7 your first term, and add or subtract it where you used 1 before, you will have the same answer.

But if we had supposed all to die at 9 years, the sum of 11 different progressions are to be subtracted: but they may all be found by multiplying the respective terms of the first, by those that die yearly; or nearly one half of these multiplications may be spared, as will appear in some following cases. You may observe here, that the numbers that die yearly go on in this order,

1, 2, 2, 6, 10, 22, 42.

$$\begin{array}{rcl}
 3 \times 342 & = & 1026 \\
 5 \times 170 & = & 850 \\
 11 \times 86 & = & 946 \\
 21 \times 42 & = & 882 \\
 43 \times 22 & = & 946 \\
 85 \times 10 & = & 850 \\
 171 \times 6 & = & 1026 \\
 341 \times 2 & = & 682 \\
 683 \times 2 & = & 1366 \\
 1365 \times 0 & = & \\
 2731 \times 1 & = & 2731
 \end{array}$$

And these added, make 11305 to be subtracted from the 21st term found at first, which would be 699051, the remainder, multiplied by 7, should be the number alive at the end of 20 years, but would differ 42 from the truth; for at the end of a full age, the increase is interrupted: The true answer is 4814264.

If you were setting down all the progressions, the first terms would always be triple those who die; and to find the sum total of a progression of this kind, double the last term, if the terms were even, and if odd, subtract the first term.

C A S E IV.

Exam. 1. I have a fine quey calf of a good breed; and which I expect to bring another at 3 years old. Now, supposing its offspring to produce at the same age, and still to bring females, how many will they amount to in 20 years? Answ. 1278.

This case is still different; and the terms here advance by always adding the last to the next before it save one.

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129.

If

If you multiply any term by 10, subtract the sixth, and add the twelfth distant, going before, you have the sixth following. So here multiply 129, the fifteenth term, by 10, subtract 13, the ninth term, and add 1, the third term, (for these are the sixth and twelfth distant preceding the fifteenth) you have the twenty-first term, or answer.

But if we had supposed all to die at 12 years, or after they had brought 10 calves, then had there been the sum of six progressions to deduce; the first of which having 8 terms, the second but 5 (for none die the 13th and 14th years.) We shall, for once, set down all these progressions, that from thence it may be understood how they are to be arranged in the other Cases.

2						
3						
4						
6	2					
9	3	2				
13	4	3	2			
19	6	4	3	4		
28.	9.	6.	4.	6.	6.	

You see the one that dies at the end of the 12th year, had she lived to the 20th, would have produced 28; the one who died at the 15th year, would have amounted to 9: The first term is double those that die; for both they and their produce are wanting that year. The sum of all these progressions is 59, to be subtracted, as being the amount of the dead, and those that would have descended from them, had they lived, and is true here; because the terms are not continued to a second age of twelve years. But these numbers are more easily obtained, as under:

$$28 \times 1 = 28$$

$$19 \times 0 =$$

$$13 \times 0 =$$

$$9 \times 1 = 9$$

$$6 \times 1 = 6$$

$$4 \times 1 = 4$$

$$3 \times 2 = 6$$

$$2 \times 3 = 6$$

These numbers are the same as before ; and observe, that three of them, viz. 9, 6 and 4, when added, are 19, the last term but one of the first progression, so they are known without the trouble of multiplying ; and the last two are equal, so that only one of them need be found, which is a great contraction, when the terms are numerous, as in the next example, where one half the multiplications are omitted. Subtract this 59, the decrease from 1278, there remains 1219, for our answer. The sum total of all the terms was not wanted here, but might be found thus : Find a term three more than enough, and from that subtract the first term, you have the total of the terms wanted.

Exam. 2. Suppose the first pair, and their descendants, to have brought a male and female every 20 years, after the age of 60, and that all lived to the age of 700 years ; how many people would be alive at the flood, accounting it 1640 years from the creation ?

This question has been taken in different senses, and we shall give the answer according to both these meanings. Supposing them to bring the first pair at 60, the second at 80, the terms will go on the same way as in the last example, a term representing the space of 20 years, and the answer in this case will be 50115859292486, males and females, young and old.

Because the terms are numerous, (for we must find the 83d term here) we will use greater multipliers, to shorten

shorten the work. If you multiply any term by 67, and add the 22d term distant before it, you have the 11th term following; or multiply any term by 4489, add double the 11th before it, and subtract the 44th, you have the 22d after it. You see the first of these multipliers cannot be used till you are past the 22d term, nor the last till after the 44th. Now set a few of the terms to begin with. 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189. Then by the rules given in the former example, find the 22d and 28th terms, and then work by these last multipliers. We give the work here at large; though in multiplying, we set down only the product.

The 16th term	189		1243524
	10		67
	1890		83316108
	17		277
22d term	1873	50th term	83316385
	10		67
	18730		5582197795
	170		18560
28th term	18560	61st term	5582216355
	67		4489
39th term	1243524		25058569217595
			166632770
			25058735850365
			277
		83d term	25058735850088

And this last term is the pairs of male and female, supposing none to have died. Now for the decrease, the same

same as in the last question, first, find a progression of 47 terms, beginning with 2, 3, 4, 6, and set them fronting one another in two columns, and multiply, setting their products also in two columns, viz. those that are multiplied by 1, (these are 4 in number); also the middle one of the other 41 progressions, in one column, the other 20 products in another column, whose sum is to be doubled, because other 20, which would be the very same, are not set down; and having subtracted the sum of all these from the 83d term, you have the answer, except what difference arises from the interruption at the end of a second period of life.

PROGRESSION.

167

	1	×	83316385	=	83316385
	0	×	56849086	=	
	0	×	38789712	=	
	1	×	26467299	=	26467299
	1	×	18059374	=	18059374
	1	×	12322413	=	12322413
1st	2	×	8407925	=	16815850
	3	×	5736961	=	17210883
	4	×	3914488	=	15657952
	6	×	2670964	=	16025784
	9	×	1822473	=	16402257
	13	×	1243524	=	16165812
	19	×	848491	=	16221329
	28	×	578949	=	16210572
	41	×	395033	=	16196353
	60	×	269542	=	16172520
	88	×	183916	=	16184608
	129	×	125491	=	16188339
	189	×	85626	=	16183314
	277	×	58425	=	16183725
	406	×	39865	=	16185190
	595	×	27201	=	16184595
	872	×	18560	=	16184320
	1278	×	12664	=	16184592
	1873	×	8641	=	16184593
	2745	×	5896	=	16184520
	4023	×	4023	=	16184529

324927108

2

156350000

649854216

156350000

806204216 decrease.

The

The decrease is here too much; for, after coming to 1400 years, another period of life is compleated; After this there is a progression of 12 terms to be managed the same way as before, and being multiplied and added, amount to 371; which must either be taken from the decrease, or added to the living.

C A S E V.

The last question, as it says, they produce after the age of 60. The meaning may be taken, That they bring the first pair only at 80, the second at 100 years old: This will give a far different answer, and the work otherwise performed; for the terms will then be had by adding always the last to the third distant preceding it, thus: 1, 1, 1, 1, 2, 3, 4, 5, 7, 10, 14, 19, 26, 36, 50, 69, 95, 131, 181, 250.

Multiply any term by 6. Subtract 5 times the fifth, and add the 15th distant going before, you have the 5th distant term following; or, multiply any term by 26, subtract 23 times the 10, add 10 times the 20th, and subtract the 30th going before, you have the 10th term following; or, multiply any term by 630, subtract 11 times the 20th, add 54 times the 40th, and subtract the 60th preceding, you have the 20th succeeding term.

PROGRESSION.

169

The 18th term	131 6.			413966 26
	786			10763116
	130			379339
	656			10383777
	1			6570
23d term	657 6	53d term	10390347 26	
	3942		270148346	
	650		9521218	
28th term	3292 6		260627128	
	19752		164930	
	3259		260792058	
33d term	16493 26	63d term	260791401 630	
	428818		164298582630	
	15111		4553626	
	413707		164294029004	
	260		35478	
43d term	413966	83d term	164294064481	

Then for the decrease raise 47 terms of a progression, as before, beginning thus: 2, 3, 4, 5, 7; and multiply as before.

1	X	5453761	=		5453761
0	X	3951206	=		
0	X	2862617	=		
0	X	2073943	=		
1	X	1502555	=		1502555
1	X	1088589	=		1088589
1	X	788674	=		788674
1	X	571388	=		571388
2	X	413966	=	827932	
3	X	299915	=	899745	
4	X	217286	=	869144	
5	X	157422	=	787110	
7	X	114051	=	798357	
10	X	82629	=	826290	
14	X	59864	=	838096	
19	X	43371	=	824049	
26	X	31422	=	816972	
36	X	22765	=	819540	
50	X	16493	=	824650	
69	X	11949	=	824481	
95	X	8657	=	822415	
131	X	6272	=	821632	
181	X	4544	=	822464	
250	X	3292	=	823000	
345	X	2385	=	822825	
476	X	1728	=	822528	
657	X	1252	=	822564	
907	X	907	=		822649
				15713794	10227616
				2	
				31427588	
				10227616	
				41655204	

The increase of the dead is here also too great by the amount of 12 terms of a progression raised and used the same way, as this, it will be 163, which subtract from the dead, or add to the living. The answer is 328504818880. However enormous these sums may be thought, one would think the suppositions moderate enough.

III. GEOMETRICAL PROGRESSION.

Numbers are said to be in geometrical progression, when they increase by the continual multiplication of a certain number, called the *ratio*; so in these numbers, 2, 4, 8, 16, 32, the number of terms is 5; the first term 2, the last 32, the ratio 2, and the sum total 63.

In an even number of these terms, the product of the first and last, is equal to the product of the two middle terms; and in an odd number, the product of the extremes is equal to the square of the mean, or middle term. To find the greatest term of any geometrical progression, add together as many of the exponents of the terms as make up the exponent of the last term; multiply all the terms together over which these exponents stand, and divide the product by the least term.

To find the sum total of all the terms, multiply the greatest term by the ratio, from the product subtract the least term, and divide the remainder by the ratio wanting one; otherwile, subtract the least term from the greatest, divide the remainder by the ratio wanting one, and to the quotient add the greatest term.

Exam. 1. What will be the produce of 1 grain of wheat for 20 years, yielding alway an hundred fold of increase?

Here

much wheat. I demand the quantity of wheat, and what acres would produce so much, at 8 quarters per acre?

0 1 2 3 4 5 6 7 8 9 10 11
1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048,
4096, 8192, 16384, &c.

The exponents, 6, 7, 8, 9, 10, 11, 12, make 63, the exponent of the 64th term; therefore multiplying these terms together, you have 9223372036854775808, the grains for the last square; and these doubled wanting 1, give the total 18446744073709551615; divide as before, you have 37529996894754 bushel, = 4691240611844 $\frac{1}{4}$ quarters. = 586406201480 $\frac{1}{2}$ acres.

Exam. 3. How many rabbits will be produced from 1 pair in 12 years, bearing at 1 year old, and 4 pairs in a year?

Answer, 488281250.—Multiply always by 5.

But if we had supposed them all to die at 6 years old, there would have been but 488125000 alive; for though only 15625 have died, these, with their increase, would have been 78120 pairs.

Exam. 4. What cost a horse, by tripling the 32 nails of his shoes with a small pin, whereof you get 32 for a halfpenny? Answer, L. 60319667605 17 1.

C H A P. XVII.

S I M P L E I N T E R E S T.

Example 1.

What is the interest of L. 724 10, for 7 years, at 4 or 5 per cent.?

At 4 per cent,

$$\begin{array}{r} 724.5 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5071.5 \\ .04 \\ \hline \end{array}$$

$$202.860 = \text{L. } 202 \text{ } 17 \text{ } 2\frac{1}{4}$$

At 5 per cent.

$$\begin{array}{r} 724.5 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5071.5 \\ .05 \\ \hline \end{array}$$

$$253.575 = \text{L. } 253 \text{ } 11 \text{ } 6$$

2. What is the interest of L. 460 16, for $7\frac{1}{2}$ years, at $3\frac{1}{2}$ or $4\frac{1}{2}$ per cent.?

at $3\frac{1}{2}$.

$$\begin{array}{r} 460.8 \\ .035 \\ \hline \end{array}$$

$$\begin{array}{r} 16.1280 \\ 7.5 \\ \hline \end{array}$$

$$120.96 = \text{L. } 120 \text{ } 19 \text{ } 2\frac{1}{4}$$

at $4\frac{1}{2}$.

$$\begin{array}{r} 460.8 \\ .045 \\ \hline \end{array}$$

$$\begin{array}{r} 20.736 \\ 7.5 \\ \hline \end{array}$$

$$155.52$$

To find the interest, at 5 per cent. you may multiply the 10th part of the principal by half the time.

3. What

3. What is the interest of L. 742 15, for $9\frac{1}{2}$ years, or for 15 years, at 5 per cent.?

for $9\frac{1}{2}$ years.

74.275

4.75

for 15 years.

74.275

7.5

$352.80625 = \text{L. } 352 \text{ } 16 \text{ } 1\frac{1}{2}$ $557.0625 = \text{L. } 557 \text{ } 1 \text{ } 3$

Or take the 100th part of the principle, and multiply it by triple the time for 3 per cent. by quadruple the time for 4 per cent. &c.

4. What is the interest of L. 125 10, for 12 years, at 3 per cent. or 4 per cent. per annum?

at 3 per cent.

1.255

36

at 4 per cent.

1.255

48

$45.180 = \text{L. } 45 \text{ } 3 \text{ } 7.$

$60.240 = \text{L. } 60 \text{ } 4 \text{ } 9\frac{1}{2}.$

5. What is the interest of L. 137 18, for 16 years, at 3 or 4 per cent.?

Answer, L. 66 3 10, at 3, and L. 88 5 $1\frac{1}{4}$, at 4 per cent.

When the time is given in days, multiply the principal by the days, divide the product by 7300, and you have the interest, at 5 per cent. and for any other rate, multiply the answer, at 5 per cent. by these decimals, by .6 for 3 per cent. by .7 for $3\frac{1}{2}$, by .8 or .9 for 4 and $4\frac{1}{2}$ per cent.

6. What is the interest of L. 500, for 40 or 50 days, at 5 per cent.?

500

40

500

50

$7300)200.00(2.739$

L. 2 14 $9\frac{1}{4}$.

$7300)250.00(3.424$

L. 3 8 $5\frac{1}{4}$

7. What

7. What is the interest of L. 90, for 60 days, at $3\frac{1}{2}$, or $4\frac{1}{2}$ per cent.?

$$\begin{array}{r} 90 \\ 60 \\ \hline 73)54.00(.7397 \\ \quad .7 \\ \hline .51779 \end{array}$$

$$\begin{array}{r} 90 \\ 60 \\ \hline 73)54.00(.7397 \\ \quad .9 \\ \hline .66573 \end{array}$$

When the sum and the days are numbers easy to manage, you may multiply a fifth part of the one by double, triple, or quadruple the other, and divide by 73, you have the interest, at 2, 3, or 4 per cent.

8. What is the interest of L. 100, for 20 days, at 2, 3, or 4 per cent.?

at 2 per cent.	at 3 per cent.	at 4 per cent.
$\begin{array}{r} 20 \\ 40 \\ \hline 73)8.00(.1095 \\ \quad 2s. 2\frac{1}{4}d. \end{array}$	$\begin{array}{r} 20 \\ 60 \\ \hline 73)12.00(.1643 \\ \quad 3s. 3\frac{1}{4}d. \end{array}$	$\begin{array}{r} 20 \\ 80 \\ \hline 73)16.00(.219 \\ \quad 4s. 4\frac{1}{2} \end{array}$

9. Borrowed of the Merchant Banking Company, May 4, L. 60, and again, June 10, L. 20 more; paid them back, Aug. 6, L. 30, and again, Oct. 4, L. 50, How much interest do I owe, at 5 per cent.?

L. Days. Products.

May 4. Received $60 \times 37 = 2220$

June 10. Received 20

Sum $80 \times 57 = 4560$

Aug. 6. Paid 30

Balance $50 \times 59 = 2950$

Oct. 4. Paid 50

$73)97.30(1.3328$ Answer.

L. 16 $7\frac{3}{4}$

10. Given in to the bank of Ayr, Jan. 1. L. 100, again, Feb. 7, L. 60, and March 9, L. 50; May 7, withdrawn L. 100, June 11, withdrawn L. 40 more; and now, July 5, I am to settle accounts, what will be due to me both of principal and interest?

Answer, L. 72 17 $\frac{1}{4}$.

	<i>L.</i>	<i>Days.</i>	<i>Products.</i>
Jan. 1. Paid in	100	$\times 38 =$	3800
Feb. 7. Ditto,	60		

	Sum	$160 \times 30 =$	4800
March 9. Paid in	50		

	Sum	$210 \times 58 =$	12180
May 7. Drawn	100		

	Balance	$110 \times 35 =$	3850
June 11. Drawn	40		

	Balance	$70 \times 24 =$	1680
--	---------	------------------	------

73)263.10(3.6014
 .8

2.88112 interest.

When partial payments are made upon bonds, at long intervals, it is usual to add the interest to the principal, before the payment be subtracted; and this is called a *progressive account*.

11. Borrowed of John Silver, upon bond, Jan. 1, 1772, L. 1000 Sterling, bearing interest, at 5 per cent. Paid back, on the 10th of March, 1773, L. 100, on the 24th of April, 1774, L. 200, on the 7th of June, 1775, L. 300, on the 6th of October, 1776, L. 150 on the 18th of August, 1777, L. 60: Now, May 1, 1778, I am to clear off this bond, what will it require to do it?

Z

1772.

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I N T E R E S T.

		L.	s.	d.
1772.				
Jan. 1.	Borrowed, upon bond, at 5 per cent.	1000	0	0
	Interest for 1 year and 69 days,	59	9	0
1773.		1059	9	0
March 10.	Paid in part of princ. and int.	100	0	0
	Balance	959	9	0
	Interest for 1 year and 45 days	53	17	8 $\frac{1}{2}$
1774.		1013	6	8 $\frac{1}{2}$
April 24.	Paid in part - - -	200	0	0
	Balance	813	6	8 $\frac{1}{2}$
	Interest for 1 year and 44 days	45	11	4 $\frac{1}{2}$
1775.		858	18	1
June 7.	Paid in part - - -	300	0	0
	Balance	558	18	1
	Interest for 1 year and 121 days	37	4	5 $\frac{1}{4}$
1776.		596	2	6 $\frac{1}{4}$
Oct. 6.	Paid in part - - -	150	0	0
	Balance	446	2	6 $\frac{1}{4}$
	Interest for 316 days - - -	19	6	2 $\frac{3}{4}$
1777.		465	8	9
Aug. 18.	Paid in part - - -	60	0	0
	Balance	405	8	9
	Interest for 256 days - - -	14	4	4
1778.				
May 1.	To be paid this day to Mr. Silver	419	13	1

Prob. 2.

Prob. 2 *To find the Discount.*

Say, As L. 100, with its interest for the time, to 100, so the given sum to the present worth; or, so the interest to the discount.

Exam. 1. What present money will discharge a debt of L. 105 6, due 7 years hence, discount being allowed, at 5 per cent. Answer, L. 78.

135 : 100 :: 105.3 : 78 or 135 : 100 :: 35 : 27.3 the discount.

2. What ready money will discharge a debt of L. 500, due 4 years hence, discount at 4 per cent.?

Answer, L. 431 0 8 $\frac{1}{4}$.

3. I have two bills upon one man, of L. 100 each, the one 6 months, the other 60 days to run: I am willing to allow discount, at 5 per cent. for ready money: What will I receive for both?

Answer, L. 196 14 1.

By the above method. But bills are often discounted by deducing the whole interest, and perhaps commission also.

4. A bill, dated Jan. 1, and payable Oct. 10, was presented June 4, and discounted by deducing interest, at 5 per cent. and $\frac{1}{2}$ commission: What does the holder receive, the contents being L. 600? Answer, $128 \times 600 = 76800$, divide by 73 = 10.5205 the interest, and 200)600(3 the commission, subtract these, there remains L. 586 9 7 $\frac{1}{4}$ to be received.

Prob. 3. *Principal, amount and time given, to find the rate.*

As the product of the principal and time to the whole interest, so is 100 to the rate per cent. or 1 to the rate of 1 pound.

Exam.

Exam. At what rate of interest will L. 60 amount to L. 92 8 in 12 years? Answer, $4\frac{1}{2}$ per cent.

Prob. 4. Principal, amount and rate given, to find the time.

Subtract the principal from the amount, the remainder divided by the product of the principal and rate, gives the time.

Exam. In what time will L. 75 amount to L. 117, at 4 per cent.? Answer, In 14 years.

Some Practical Questions.

Quest. 1. There was laid into the bank, for my behoof, L. 150, when I was born, bearing interest, at 4 per cent. and just now I discounted them a bill of L. 292, 100 days to run, at 5 per cent. which exactly clears them and me: What is my age?

Answer 23 years, 3 days.

2. Given the principal L. 40, and the amount L. 55, and the sum of the time and rate added together, viz. 9.415, and the time and rate severally required?

Answer, The rate 4 per cent. the time 9 years, 136 days, 21 hours.

Divide the total interest by the principal, the quotient is the product of the rate and time.—*Then see prob. 6th of Extraction of Roots.*

3. Received, Aug. 1. 1778, the contents of two bills of L. 60 each, bearing interest from their date, the one at 5, and the other at 4 per cent. and the one amounting to L. 5 2 more than the other: Can you tell me the date of these bills?

Answer, Feb. 1, 1770.

4. Paid L. 10 Sterling of rent, seed, labour and manure, for an acre of land the first year, and L. 2 for each

each of the two following years: How many bolls of grain must it produce one year with another, at 20s. per boll, to make me cent. per cent. of my money laid out? Answer 9 bolls, 9 pecks, $1\frac{3}{8}$ lippies.

Cent. per cent. for the money will be found to be L. 34 4 for a dividend, then 3 crops with $\frac{1}{10}$ interest or $\frac{6}{10}$ for a divisor, gives as above, and for proof, multiply this by 3, and add 3 years interest, viz. 2 years interest of one crop, and 1 of another.

II. COMPOUND INTEREST.

Prob. 1. *Principal, rate and time given, to find the amount.*

The principal is the first term of a Geometrical Progression, the amount of L. 1 for a year the ratio; wherefore multiply as often as there are years, by 1.05 for 5 per cent. by 1.04 for 4 per cent. The work may be contracted as you see here.

Exam. 1. What will L. 20 amount to in 7 years, at 4 or 5 per cent. compound interest?

at

	at 4 per cent.	at 5 per cent.
	20	20
	.80	1
<i>Years.</i>		
1.	20.80	21
	.8320	1.05
2.	21.6320	22.05
	.8652	1.1025
3.	22.4972	23.1525
	.8998	1.1576
4.	23.3970	24.3101
	.9358	1.2155
5.	24.3328	25.5256
	.9733	1.2762
6.	25.3061	26.8018
	1.0122	1.3400
7.	26.3183	28.1418
	or L. 26 6 $\frac{1}{4}$.	or L. 28 2 10.

The principal taken from the amount, leaves the interest.

To find the amount for any part of a year, multiply the principal by that root of the ratio (that is, the amount of L. 1 for a year) whose index expresses that part, viz. for $\frac{1}{2}$ year extract the square root, for $\frac{1}{4}$ the biquadrate, &c.

2. What is the interest of L. 354 16, for $3\frac{1}{2}$ years, at 5 per cent. compound interest?

Answer, L. 66 1 $\frac{1}{4}$.

The

The amount for 3 years is 410.7201, which multiplied by 1.02469, the square root of 1.05, produces 420.86, &c. from which take the principal.—*These questions are easily answered by the Tables.*

3. What is the interest of L. 250 10, for 20 years, at 4 or $4\frac{1}{2}$ per cent. per annum, compound interest?
 Anfw. L. 298 7 $6\frac{1}{4}$, at 4 per cent. or L. 353 12 8, at $4\frac{1}{2}$.

Under 4 per cent. and opposite to 20 years in Table I. you find 2.1911231; which multiply by the given principal, you have the amount, at 4 per cent.

If the interest be sought for years and quarters, find the amount for the years, which multiply by the number in Table VI. opposite to the quarters.

Prob. 2. *Amount, rate and time given, to find the principal.*

Find the amount of L. 1 for the time, and rate given, and say, As that amount is to 1, so is the given amount to the principal sought.

Or divide the given amount by the number found in the first Table, under the rate, and opposite to the time; or multiply by that found in the second, and the quotient, or product, is the number sought.

Exam. 1. What ready money will discharge a debt of L. 25 10 6.606, or L. 25.52563125, due 5 years hence, discounting at 5 per cent. compound interest?

Answer, L. 20.

2. What ready money will discharge a debt of L. 10000, due 15 years hence, discount being allowed at $4\frac{1}{2}$ per cent. compound interest?

Answer, L. 5167 4 1 nearly.

Prob. 3.

Prob. 3. *Principal, amount and time given, to find the rate.*

Divide the amount by the principal, extract that root of the quotient, whose index is the number of years, and from the root take 1.

Or with the same quotient in your mind, go in a line from the years, till you find it in Table I. and at the head of the column you have the rate.

Exam. At what rate of interest will L. 40 amount to L. 46.7943424 in 4 years?

Answer, At 4 per cent. compound interest.

Prob. 4. *Principal, amount and rate given, to find the time.*

Find the quotient of the amount by the principal, which divide continually by the amount of L. 1 for a year, till the last quotient be 1, and the number of divisions will be the number of years.

Or look under the rate, in Table I. till you find the same quotient, opposite to which, on the left hand, you find the years.

Exam. 1. In what time will L. 500 amount to L. 562.432, at 4 per cent. compound interest?

Answer 3 years.

2. In what time will L. 100 amount to L. 551 12, at 5 per cent. compound interest?

Answer 35 years nearly.

3. In what time will L. 1000 amount to L. 1947 18, at 4 per cent. per annum, compound interest?

Answer, In 17 years.

C H A P. XVIII.

I. ANNUITIES IN ARREAR.

Case 1. *Annuity, time and rate given, to find the amount.*

Find such a principal, as will yield as much interest yearly, as is the annuity; find the amount of this principal, for the proposed time, by the first problem of Compound Interest; from which subtract the said principal, there remains the amount of the annuity.

Or multiply the number found under the rate, and opposite to the time in Table III. by the Annuity, the product is the answer.

Exam. 1. If an annuity of L. 45 per annum, be forborne 5 years, what will then be due, compound interest being computed at 5 per cent.?

Answer, L. 248 13 $0\frac{3}{4}$.

2. What will an annuity of L. 80 per annum, amount to in 30 years, compound interest being reckoned at 4 per cent.? Answer, L. 4486 15 11.

For any part of a year, as a quarter, multiply by the proportional number in Table VI. and add the part of the annuity.

Case 2. *Amount, rate and time given, to find the annuity.*

Find the interest of L. 1 for the time, and at the rate given, by which divide the given amount, the quotient is the corresponding principal, 1 year's interest of which is the annuity.

Or divide the given amount by that found in Table III. even with the years, and under the rate.

A a

Examples.

Examples.

1. What annuity will amount to L. 248.65340625, in 5 years time, at 5 per cent. compound interest?

Answer, L. 45.

2. What annuity, to continue 25 years, will amount to L. 1000, compound interest at 3 per cent.?

Answer, L. 27 14 $0\frac{1}{2}$.

Case 3. *Annuity, amount and rate given, to find the time.*

Find the corresponding principal, and adding this to the amount of the annuity, work by prob. 4th of Compound Interest.

Or divide the amount by the annuity, the quotient is the amount of L. 1 annuity, and seek for this under the rate in Table III. you have the time on the left hand.

Exam. 1. In what time will an annuity of L. 3 15 per annum, amount to L. 25 10 $1\frac{1}{2}$, at 5 per cent.?

Answer, 6 years.

2. In what time will an annuity of L. 25 a-year, amount to L. 2675 15 2, at $4\frac{1}{2}$ per cent. compound interest? Answer, In 40 years.

Case 4. *Annuity, amount and time given, to find the rate.*

Divide the amount by the annuity, and seek the quotient in Table III. even with the time.

Exam. 1. At what rate of interest will L. 75 of annuity amount to L. 422.781972 in 5 years?

Answer, 6 per cent.

If you subtract the annuity, and find all the divisors of the remainder, one of them is 106, whereby we know the rate to be 6 per cent.

2. At what rate of interest will L. 50 amount to L. 215.50625 in 4 years? Answer, 5 per cent.

II. OF

II. OF PURCHASING ANNUITIES.

Case 1. *Annuity, time and rate given, to find the present worth.*

Find a principal, of which find the present worth, as a sum due at the end of the time, subtract that present worth from the principal, the remainder is the worth of the annuity.

Or multiply the number in Table IV. at the time and rate proposed, by the given annuity.

Exam. 1. What is the present worth of an annuity of L. 60 per ann. to continue 6 years, at 4 per cent.?

Answer, L. 314.528214.

2. What is an annuity of L. 100 per ann. to continue 25 years, worth in ready money, accounting interest at $4\frac{1}{2}$ per cent.? Answer, L. 1482 16 5,

Case 2. *Present worth, time and rate given, to find the annuity.*

Find the present worth of L. 1 annuity, by which divide the present worth given.

Or multiply by the number in Table V. answering the rate and time.

Example. What annuity, to continue 5 years, will L. 100 purchase, at 5 per cent.? Answ. L. 23.0974 +

Case 3. *Annuity, present worth and rate given, to find the time.*

Add the present worth and annuity together, from that sum subtract the product of the present worth, multiplied by the amount of L. 1 for a year, divide the annuity by this remainder, the quotient will be such a power of the amount of L. 1 as is denoted by the

the number of years, and which continually divided by the amount of L. 1 till the quotient come to be 1, the number of divisions will be the years.

Or divide the present worth by the annuity, and search for the quotient in Table IV. answering the rate.

Exam. 1. Purchased an annuity of L. 30 per ann. at 4 per cent. and paid down L. 133 11 1: How long must the said annuity continue?

Answer, 5 years.

2. Purchased an annuity of L. 50 a-year, at $4\frac{1}{2}$ per cent. and paid L. 920.07922: What time will this annuity continue? Answer, 40 years.

Case 4. *Annuity, present worth and time given, to find the rate.*

Divide the worth by the annuity, and seek for the quotient in Table IV. even with the time, and you will find it under the rate.

Exam. Purchased an annuity of L. 60, to continue 40 years, and paid L. 1187.566326: What rate of interest had I for my money? Answer 4, per cent.

III. ANNUITIES IN REVERSION.

Case 1. *Annuity, time and rate given, to find the value.*

Find the present worth of the annuity, as if to commence just now, by case 1. Section II. then find what principal will amount to that sum, by the time the annuity commences, by the 2d problem of Compound Interest.

Exam. A farmer has 4 years to come of a lease, worth L. 32 per annum, and is desirous to have 7 years more added to the lease, and will pay ready money, at 5 per cent. What sum ought he to pay?

Answer, L. 152.3348.

By

By the Tables thus: Seek in Table IV. under 5 per cent. for the number opposite to 11 years, viz. the whole time, both in being and reversion; from this subtract the number even with 4 years, that is, the time in being, and multiply the remainder by the annuity, viz. 32, gives the present worth.

Case 2. Present worth, time and rate given, to find the annuity.

Find the amount of the present worth for the time in being, by prob. 1. of Compound Interest, and find what annuity that amount will purchase, for the time of the reversion, by case 2. of Section II.

Exam. Purchased an annuity, to continue 8 years, and to commence 2 years after purchase, and being allowed 4 per cent. I paid down L. 622.48: I demand the annuity? Answer, L. 100.

Case 3. Annuity, worth and rate given, to find the time.

Find the amount of the worth, for the time in being, and taking that amount, for the worth, work as in case 3. of Section II.

Exam. Purchased an annuity of L. 20 per ann. to commence after 2 years, and being allowed 5 per cent. paid down L. 64.326: How long ought the annuity to continue? Answer, 4 years.

IV. FREEHOLD ESTATES.

Case 1. Rent and rate of interest given, to find the value, or price.

Divide the yearly rent by the rate of L. 1 for a year.

Exam. An estate of L. 500 yearly rent, is to be sold, what is it worth, allowing the purchaser 5 per cent. for his money? Answer, L. 10000.

Case 2. Value and rent given, to find the rate of interest.

Divide the rent by the value, quotes the rate of L. 1.

Exam. An estate of L. 70 per annum, is sold for L. 1750, what rate of interest hath the buyer for his money? Answer, 4 per cent.

Case 3. When value and rate are given, to find the rent.

Multiply the value by the rate of L. 1.

Exam. Bought an estate, at L. 1500, at 4 per cent. What is the rent? Answer, L. 60.

If the value of an estate is not told, but is said to be at so many years, divide 1 by the number of years purchase, gives the rate.

Exam. Bought an estate, at 25 years purchase: What rate of interest had I for my money?

Answer, 4 per cent.

Or if the years purchase be required, divide 1 by the rate of L. 1.

Exam. Bought a house, and am allowed $6\frac{1}{3}$ per cent. for my money: How many years purchase did I give for the house? Answer, 15 years.

V. FREEHOLD ESTATES IN REVERSION.

Case 1. The yearly rent and rate of interest given, to find the value of any freehold estate in reversion.

Find the absolute value of the estate, and, by the rules of Compound Interest, find the present worth of that value, as a sum due so many years hence, as the time in being.

Or otherwise, thus: Find the present value of the rent as an annuity for the time in being, and subtract this from the value of the estate.

Exam. Suppose an estate of L. 100 per annum is to be sold just now, but not to be entered upon for 7 years: What is it worth in ready money, allowing the purchaser 5 per cent.

Answer, L. 1421 7 3.

Case 2. Present value, rate of interest and time in being given, to find the rent.

Find the amount of the present worth for the time in being, then find the income that amount will purchase, by case 3d of a former section.

Exam. Paid L. 1000 for the reversion of an estate, which I am not to enter upon possession of for 5 years: What yearly rent must I let this estate at, so as to have 5 per cent. for my money?

Answer, L. 63 16 10 $\frac{1}{2}$ nearly.

Case 3. Present worth, yearly rent, and rate of interest given, and the time in being required.

Find the absolute value of the estate, and also in what time the present value will amount to that absolute value.

Exam. Purchased an estate of L. 50 per ann. from an old man, and was to take possession of the same at his decease; which accordingly I did, and find I have made 5 per cent. of the money I paid him, which was L. 710.6813: I demand how long the old man had lived after our bargain? Answer, 7 years.

Case 4. Rent, value and time in being, given, to find the rate of interest.

No absolute value of the estate can be found, without condescending upon some rate of interest; but we shall propose a question, in which interest may be supposed on the money, without finding the value of the estate:

Exam.

Exam. I made a foolish bargain at my marriage, in purchasing an estate of L. 500 a-year; for which I paid, in ready money, the sum of L. 10000, on condition that I should enter upon possession whenever my wife bare a man-child: As ill-luck would have it, she had no male-issue for 10 years. Now what rate of interest have I for my money?

It is to be observed, I might have had 4 per cent. for it all these 10 years.

If the L. 10000, that lies dead for 10 years, had been put out to interest, at 4 per cent., it would now have amounted to L. 14802.443 (for we reckon compound interest here); and this is what the estate costs him, when he enters upon possession, for which he receives only L. 500 a-year; that is less than $3\frac{1}{2}$ per cent. being only L. 3 7 $\frac{1}{2}$.

VI. QUESTIONS, to EXERCISE the RULES in the two foregoing Chapters, and following Tables.

Quest. 1. Lent in 1768 per bill, at 5 per cent. L. 100, and got the bill renewed at the end of every two years, and the interest added to the principal: What will the contents of it be at the end of the year, 1788?

Answer, L. 259 7 $\frac{5}{4}$.

Work for half the time, at double the rate.

2. A would sell an annuity, for 15 years, of L. 50 per annum; for which B offers him L. 500, ready money; C, L. 700, at the end of 7 years; and D would give him L. 400, in hand; and the reversion of an annuity of L. 20 per annum, to begin after 7 years, and to continue 8 years: Which offer is the best?

Answer, B's best, by L. 2 10 $\frac{5}{2}$; D's worst, by L. 5 12 $\frac{2}{3}$.

3. A has

3. A has an annuity of L. 60, per annum, to continue 24 years; B has the reversion of an annuity of L. 100 per annum, to commence after 5, and to continue 20 years; and C has a freehold estate of L. 40 per annum; D the reversion of one of L. 70 per annum, to be entered to after 6 years: These four gentlemen sell their properties, all at one bargain, and receive L. 3000; which they divide at present, not according to their right, but as each has occasion: So A takes L. 1200, B, L. 800, C, L. 600, and D, L. 400; after 4 years they come to clear accounts: What will be due by each to the other, computing simple interest on the balances, at 5 per cent.

Ans. The present worths, at 5 per cent. are thus: Of A's annuity L. 827.918508, of B's, L. 976.44678, of C's estate L. 800, of D's, L. 1044.70156.

Then proportioning L. 3000 according to these present worths, A should have had only L. 680.6549, B, L. 802.7642, C, L. 657.7023, and D, L. 858.8781: So that A had got L. 519.3451 more than his share, B wanted L. 2.7642 of his, C, L. 57.7023, and D, L. 458.8781: Therefore, accounting simple interest at 5 per cent. A will have to pay L. 623 4 3 nearly, B will receive L. 3 6 4, C, L. 69 4 10 $\frac{1}{4}$, and D, L. 550 13 0 $\frac{3}{4}$.

4. I made a bargain with an old man, that for L. 100 in hand, I would, after 7 years, pay him L. 100 yearly while he lived: How long must the old man enjoy this annuity, to be even with me?

Answer, 1 year and 82 days only, at 5 per cent.

5. There is an annuity of L. 30 per annum run 20 years in arrear: What annuity, to continue 20 years, will pay up this arrear?

Answer, L. 65.7337, at 4 per cent. or L. 79.5989, at 5 per cent.

6. A farmer is offered a lease of lands for 19 years, at L. 36 yearly rent, and L. 200 of *fine grassum*, or entry-money;

B b

entry-money;

try-money; but he would rather chuse to pay no entry-money, to which the proprietor agrees: How much yearly rent must he pay in that case, so as to make a bargain equally good with the first offer?

Ans^r. L. 51 4 $6\frac{1}{2}$, at 4 per cent. or L. 52 10 $11\frac{1}{4}$, at 5 per cent. nearly.

7. A farmer offers L. 40 of yearly rent for a lease of lands, for 19 years, and L. 60 of fine; but the proprietor proposes to abate a reasonable part of the rent, provided he will pay L. 100 of fine: What must the rent be in this case?

Answer, L. 36 19 1 nearly, at 4 per cent. or L. 36 13 $9\frac{1}{2}$, at 5 per cent.

8. A certain person has left him a legacy of L. 1000, due 4 years hence; and at the same time, he owes the executor an annuity of L. 20 per annum for 12 years, and which is now 8 years in arrear; they resolve to make a clearance just now: What will be the balance between them, discounting the legacy by simple interest?

Answer, The executor must pay the legatee L. 605 3 $8\frac{1}{2}$, at 4 per cent. or L. 571 8 $7\frac{1}{2}$, at 5 per cent.

9. A has a lease for 12 years, of a victual-farm, L. 70 yearly rent; B has a lease of a store-farm, L. 100 yearly rent, for 8 years: Now, these two want to make an exchange of their possessions, what is the balance, and by whom payable?

Answer, A pays L. 16 6 $4\frac{1}{2}$, at 4 per cent. or, L. 25 17 $10\frac{1}{2}$, at 5 per cent.

10. A farmer, who has just now taken a lease of his ground for 19 years, paying yearly rent L. 45, and L. 80 gratuity, proposes to take a reversion of the same for 19 years more; What fine must he pay for this last lease, so as to make a bargain equally good with the first?

Answer, L. 37 19 5, at 4, or L. 31 13 2, at 5 per cent.

11. A

11. A delivers to B an annuity of L. 40 per ann. to commence just now; B assigns to A his right to another annuity of L. 60 per annum, but which does not commence for 5 years; they are to continue in possession of these till they are clear of each other's debt, and no longer: I demand how long this bargain will be in force?

Answ. 16 years, 126 days after the commencement of last annuity, or 21 years, 126 days from making the bargain.

Find the amount of L. 40 for 5 years, and then how long an annuity of L. 20 will take to discharge that debt. We wrought here at 5 per cent.

12. A has an annuity of L. 20 for 12 years in all, and which is now 5 years in arrear; B has the reversion of an annuity of L. 40, to commence after 5, and to continue 15 years: Now, these two men would change annuities; what must one pay the other as a balance?

Answ. A pays L. 99 1 4 $\frac{1}{2}$, at 5, or L. 137 3 5 $\frac{1}{4}$, at 4 per cent.

13. A lends to B L. 1000; B gives A an annuity of L. 20 per annum, commencing just now, and the reversion of another of 50 per ann. after 4 years: Now, at 12 years end they come to settle accounts, who will be debtor, and how much, compound interest being reckoned on both sides, at 5 per cent.?

Answ. B owes A the L. 1000 still, and 14d.

14. A and B receive, in a legacy equally between them, an annuity of L. 100, to continue; they agree, that A shall enjoy the same a certain number of years, at present, and B the reversion thereof: How long must each enjoy it, so as to have a just share in the legacy?

Answ. A, 8 years, 29 days, 4 hours, 48 minutes, B, 11 years, 335 days, 19 hours, at 4 per cent. or A, 7 years,

years, 237 days, 6 hours, B, 12 years, 127 days, 18 hours, at 5 per cent.

15. Four tenants, A, B, C and D, whose farms lie contiguous to one another, and whose tacks are nearly expired, send one of their number to the proprietor, to get their leases renewed: He accordingly pays L. 1000 of *grassum*, for a lease of 19 years, just now commencing: I demand how much of that sum each of the farmers must pay, there being 2 years of A's old tack to run, 3 of B's, 4 of C's, and 5 of D's; also A's yearly rent is L. 40, B's L. 50, C's L. 60, and D's L. 70?

Answer, The present worths of their leases, considering them as annuities in reversion, at 5 per cent. are as follow: A's L. 409.0364, B's L. 468.1036, C's L. 512.3622, and D's 542.9090; and then proportioning L. 1000 by these present worths, A pays L. 211 13 5, B, L. 242 4 9 $\frac{1}{4}$, C, L. 265 2 10, and D, L. 280 18 11 $\frac{1}{4}$.

16. A has an annuity of L. 100 per annum for 7 years; B has the reversion of it for 7 years more; and C a further reversion of it for other 7 years. Being in need of money in hand, they agreed to sell their rights to it, and are bid off-hand the round sum of L. 1000; which they accept: How will this money be divided amongst them?

Answer, A takes L. 451 6 3 $\frac{1}{2}$, B, L. 320 14 9 $\frac{1}{4}$, C, L. 227 18 10 $\frac{3}{4}$.

17. A youth of 15 years, has an annuity left him of L. 500 per annum, and which is to run in arrear till he is 21 years of age; at which time he lifts off the arrears L. 1000, and lets it run in arrear 6 years longer, when he receives L. 1000 more; after which it is let alone to the end of the time of the annuity, which is also 6 years, and he gets another L. 1000; and 6 years after that again is for a settlement: *Quæritur*, What will now clear him off, compound interest, at 5 per cent.?

Answer,

Answer, The amount of the annuity, for 6 years, is L. 3400.9564; subtract L. 1000, there is left L. 2400.9564; the amount of this, for 18 years, is L. 5778.18777, this is so much he has to receive. The amount of the annuity, for 6 years more, is, as above, L. 3400.9 &c. and subtracting L. 1000, there is left L. 2400.9; and this, in 12 years, amounts to L. 4311.7726, so much more he has to receive; and of the last 6 years of the annuity, the amount still the same, subtracting the last L. 1000, the remainder in 6 years, amounts to L. 3217.111, and this is another sum he has to receive. Add these three sums together, and you will find it to require L. 13307 9 5, to clear off the annuitant. Or this question might be answered thus:

Find the amount of the annuity for 18 years, viz. L. 14066.19235; find the amount of this as a principal for 6 years, which will be L. 18850.04246; then find the amount of L. 1000 for 18 years, 12 years, and 6 years, and all these three added together, make L. 5542.5711; subtract this from the former amount, rests L. 13307.47136, as before.

18. An old man, who has an estate of L. 50 per annum, would dispose of it for an annuity, to continue 10 years, which he thinks is as long as he shall live: I demand what the annuity will be per annum?

Answer, L. 154 2 $3\frac{1}{4}$, at 4, or L. 129 10 1, at 5 per cent.

19. A has a small estate in reversion of L. 90 per annum, and B has the reversion of an annuity of L. 120 per annum, for 20 years, the time in being is 7 years with both; they make an exchange, and the balance passes to account till 27 years are ended: What will then be due, and by whom?

Answer, At 5 per cent. the present worth of A's estate is L. 1279.22634, and of B's annuity L. 1062.79924; so the balance is due by B, and which in 27 years will amount to L. 808 0 5.

20. A gentleman left to his three sons, *This*, *That* and *The other*, a freehold estate of L. 500 per annum equally amongst them; they agree to enjoy it in manner following, viz. *This* shall possess it for a certain time, *That* the reversion of it for another certain time, and *The other* the reversion of it for ever after: Now, how long must *This* and *That* possess it, so as all three may share alike in the legacy?

Ans. *This* must enjoy the estate 8.31 years, *That* 14.207 years, at 5 per cent.

21. An indolent person has left him, in a legacy, an annuity of L. 40 per ann. to continue for 20 years, with the burden of paying L. 500 of debt. He lets the annuity run in arrear, and none of the debt he pays; and after 7 years, has another annuity of L. 25 left him, to continue 12 years, and incumbered with L. 200 of debt; however, he behaves with this as with the former, and, after 6 years, dies himself, leaving all to his heir, with the expence of his funeral, which was L. 25: What is the heir benefited by his death?

Answer, Only 21s. 4 $\frac{3}{4}$ d.

The amount of the first annuity, for 13 years, and the present worth of the same, for 7 years more, is L. 939.974248, or the present worth of the annuity, for 20 years, and the amount of that present worth, for 13 years, is the same; but the amount of L. 500, for 13 years, is L. 942.8245: So there is a loss here of L. 2.850302. The amount of the present worth of the other annuity, for 6 years, is L. 296.94012, and the amount of L. 200 of debt, for 6 years, is L. 268.01912: So there is here a profit of L. 28.921; from which take the former loss, and the funeral charges, you will find as above, at 5 per cent.

22. Purchased two farms, the one at 4, the other at 5 per cent. and gave L. 1000 more for the one than for the other: The yearly rent is required, being the same in both? Answer, L. 200 each.

Divide

Divide the difference of the prices by the difference of years purchase.

23. Bought, this present year 1789, two estates in reversion, for which paid L. 17500 8 5 1.216 qrs. the rent the same in both, viz. L. 600; but I bought the one at 4, the other at 5 per cent. and so gave L. 2766 10 0 3.264 qrs. more for the one than for the other: When am I to enter upon possession?

Answer, In 1799.

The sum and difference of their values being given, the value of each is known.—*Then see sect. V. case 3. of this chapter.*

24. Sold an estate of L. 500 a-year, at 5 per cent. and the reversion of another, after my decease, of the same rent, for L. 3768.895: How long must my life be prolonged, to render this last an equal good bargain with the first? Answer, 20 years.

25. Bought two estates in reversion, the rent of each L. 209 0 4; and as I bought the one at 4, the other at 5 per cent. I paid L. 1000 more for the one than for the other; I am to enter upon possession when I am 21 years of age: Can you tell me what my age is now?

Answer, 14 years of age; or it is 7 years before you enter upon possession, which is what we have to find out here.

This question is somewhat difficult, but may be resolved as follows: Find the difference of the absolute values of the estates, viz. L. 1045.083; from which take the difference of the values of the reversions L. 1000, rests L. 45.083: This is the difference of the present worths of two annuities equal to the rent, for the time in being, at the different rates per cent. wherefore divide 45.083 by the rent, the quotient .2156, &c. is the difference of L. 1 annuity for these rates, and will be found to be the difference of the numbers

numbers in Table IV. under 4 and 5 per cent. and opposite to 7 years.

26. A had an estate of L. 560 a-year, B one of L. 600, and C one of L. 800; all three sold their estates to one person, at 5 per cent. and bargained for ready money; however, after wanting it 3 years, the purchaser becomes bankrupt, and pays only half a crown in the pound: How much is each a loser?

Ans. A loses L. 11344 14 6, B, L. 12155 1 6, C, L. 16206 15.

27. I have an estate of L. 500 a-year, burdened with the payment of L. 500 to my brother, at 21 years of age, and who is now 16; also L. 150 to each of my 3 sisters, 6, 8 and 10 years hence: I would sell my estate just now with these incumbrances, how much money will it produce, at 25 years purchase?

Answer, L. 11759 11 0 $\frac{3}{4}$ nearly, at 4 per cent.

28. A farm, which is set in tack for 21 years, pays L. 100 yearly rent for the first 7 years, L. 200 for the next 7, and for the last 7, L. 300 yearly: This farm is just now to be sold, what will it bring?

Answer, L. 4548 1 8, at 4, or L. 3553 5 2 $\frac{1}{4}$, at 5 per cent.

29. Lent, at 5 per cent. L. 400, and at the same time borrowed, at 4 per cent. L. 490: When will the amount of the first sum pay that of the second?

Answer, In 225 years simple interest.

Divide 90, the difference of the principals, by .4, the difference of their interests for a year. At compound interest it would take about 21 $\frac{1}{2}$ years. Carry on the progression of the amount of both sums till you find them agree.

30. A pays L. 1800 for an annuity of L. 100 per annum, to continue 40 years; B puts out L. 1800 to interest at the same time: Which amount will be greatest at the expiration?

Answer,

Answer, B's will exceed by L. 574. The present worth of which is L. 81 10 $8\frac{1}{4}$, at 5 per cent.

31. A certain person has owing him L. 4204.04655, due at the expiration of 40 years; but agrees to take value at 10 equal payments, viz. at every 4 years end: What will one of these payments be, reckoning interest at 5 per cent.? Answer, L. 150.

Take the numbers in Table I. opposite to the years 4, 8, 12, 16, 20, 24, 28, 32, 36, under 5 per cent. and with the sum of all these plus 1, divide the sum given.

32. What annuity, to continue 40 years, will pay a debt of L. 145000000, at 4 per cent.?

Answer, L. 7325907 10.

From which take L. 5800000, the yearly interest, the remainder, viz. L. 1525907 10, will be a sinking fund, to pay off L. 145000000 in 40 years.

C H A P. XIX.

C U R I O S I T I E S.

Prob. 1. *How to resolve an account of Addition, before it be set down, saving only the first line.*

When the head-line is written, ask any person, how many lines more are to be set down, one half of which he is to set down, and you the other; subtract the number each are to write, from the head-line, and set the remainder at a proper distance, and the same number you subtract on the left hand, this will be the sum total; then let him set down his lines, and in setting of yours, subtract every figure of his from 9. This may also be applied to divers denominations.

C c

Prob. 2.

Prob. 2. *To find the number one shall think of.*

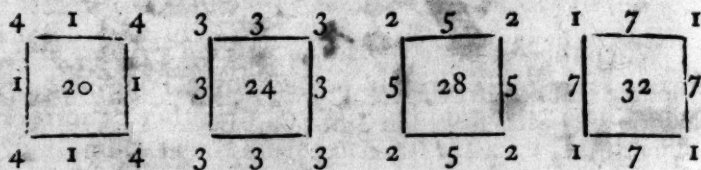
When one has thought of a number, bid him triple it; ask him, whether it be odd or even; if he say, odd, bid him add 1, and for this keep 1 in your mind; bid him throw away one-half, and triple the other half, asking again, if odd or even; if he say, odd, bid him add 1, and for this reserve 2 in your mind; then bid him throw away one-half, and from the other bid him take 9, 18, 27, or any number of 9s, so you will discover how many 9s there are, and for every 9 take 4, and add to those reserved in your mind.

Prob. 3. *To tell one a number he has in mind without asking any thing.*

When any one has thought of a number, bid him double it, then bid him add any even number you please, then bid him throw away one half, and also the number he thought on first, and you tell him how many remains, which will always be half the number you bade him add.

Prob. 4. A family, that consisted sometimes of 20, 24, or 28, and sometimes of 32 persons, always found a way to place themselves on the four sides of a square room, so as to be 9 on each side: How could that be?

See below:



Prob. 5. *To discover hidden things.*

Take 24 halfpennies, and lay them on the table; give 1 of them to your first man, 2 to the second, and

and 3 to the third; then let there be 3 different things which they are to dispose of among them, unknown to you, and which we shall call *A*, *B* and *C*. When you are withdrawn, bid him that has *A*, lift as many halfpennies from the table as you gave him, him that has *B* twice as many, and him that has *C* four times as many; then come forward to the table, and observe how many halfpennies remain; if there is only 1, your first man has *A*, your second *B*, and your third *C*; if 2 are left, the first has *B*, the second *A*, and the third *C*, as in the following table:

$$\begin{array}{ccccccc}
 1 \left\{ \begin{array}{l} 1 A \\ 2 B \\ 3 C \end{array} \right. & 2 \left\{ \begin{array}{l} 1 B \\ 2 A \\ 3 C \end{array} \right. & 3 \left\{ \begin{array}{l} 1 A \\ 2 C \\ 3 B \end{array} \right. & 5 \left\{ \begin{array}{l} 1 B \\ 2 C \\ 3 A \end{array} \right. & 6 \left\{ \begin{array}{l} 1 C \\ 2 A \\ 3 B \end{array} \right. & 7 \left\{ \begin{array}{l} 1 C \\ 2 B \\ 3 A \end{array} \right.
 \end{array}$$

Prob. 6. *To know the card one shall think of.*

Take any odd number of cards, that is divisible by 3, lay them in 3 heaps three times, always asking the person, which heap his cards is in, and always putting that heap middlemost; so his card will be the middlemost of the whole number, the last time you put them up.

Prob. 7. *To dispose of the numbers, from 1 to 25 inclusive, in 5 lines, so as, adding them every way, you may still have 65.*

1	14	22	23	5
18	7	20	9	11
10	24	13	2	16
15	17	6	19	8
21	3	4	12	25

Prob. 8.

Prob. 8. *To dispose of the numbers, from 1 to 49 inclusive, in 7 lines, so that, adding them every way, you may still have the same number, viz. 175.*

33	44	38	18	6	5	31
8	41	10	11	47	37	21
15	16	49	4	43	34	14
28	24	23	25	26	22	27
30	35	7	46	1	20	36
42	13	3	39	40	9	29
19	2	45	32	12	48	17

Prob. 9. *To find a perfect number.*

A perfect number is one, all the divisors whereof added make the number, and to find which, let the progression 1, 2, 4, 8, be continued to 2, 3, 5, 7, or any prime number of terms, and multiply the sum of all these by the last term, and the product will be a perfect number. Such numbers are, 6, 28, 496, &c.

Prob. 10. *To find all the just divisors of any number.*

Make as many columns as the number has component primes, and place the primes at the heads of the columns. When a prime is several times repeated, set its powers in the same column; then multiply each number in the first column by every one in the second, setting all the products in the second column; and do the same with the other columns.

T A B L E I.

The amount of L. 1 for years, compound interest.

Y.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000
2	1.1090000	1.0712250	1.0860000	1.0920250	1.1025000
3	1.0927270	1.1087178	1.1248640	1.1411661	1.1576250
4	1.1255088	1.1475230	1.1698585	1.1925186	1.2155002
5	1.1592740	1.1876863	1.2166529	1.2461819	1.2762815
6	1.1940522	1.2292553	1.2553190	1.3022601	1.3400956
7	1.2298738	1.2722792	1.3159317	1.3608618	1.4071004
8	1.2667700	1.3168090	1.3685690	1.4221006	1.4774554
9	1.3047731	1.3628973	1.4233118	1.4860951	1.553282
10	1.3439163	1.4105987	1.4802442	1.5529694	1.6288946
11	1.3842338	1.4595540	1.5395440	1.6228530	1.7103393
12	1.4257608	1.5110686	1.6010322	1.6958814	1.7958563
13	1.4685337	1.5639560	1.6650735	1.7721961	1.8856491
14	1.5125898	1.6186945	1.7316764	1.859449	1.9799316
15	1.5579674	1.6752488	1.8009435	1.9352824	2.0789281
16	1.6047064	1.7339860	1.8729812	2.022370	2.1828745
17	1.6528476	1.7940755	1.9479005	2.1133768	2.2920183
18	1.7024330	1.8574892	2.0258165	2.2084787	2.4066192
19	1.7535060	1.9225013	2.1068491	2.3078603	2.5269502
20	1.8061112	1.9897888	2.1911231	2.4117140	2.6532977
21	1.8602945	2.0591314	2.2787080	2.5202411	2.7859625
22	1.9161034	2.1315115	2.3699187	2.6336520	2.9252607
23	1.9735865	2.2061144	2.4647155	2.7521663	3.0715237
24	2.0327941	2.2833284	2.5633041	2.8760138	3.2250999
25	2.0937779	2.3632449	2.6658363	3.0054344	3.3803549
26	2.1569912	2.4459585	2.7724697	3.1406790	3.5550726
27	2.2212890	2.5315671	2.8833685	3.2820095	3.7334563
28	2.2879762	2.6201719	2.9987033	3.4296999	3.920291
29	2.3563655	2.7118779	3.1186514	3.5840364	4.1161356
30	2.4272624	2.8067937	3.2433975	3.7453181	4.3219423
31	2.500080	2.9050314	3.3731314	3.9138574	4.5380394
32	2.5750827	3.0067075	3.5080587	4.0899810	4.7649414
33	2.6523352	3.1119423	3.6483811	4.2740301	5.0031885
34	2.7319053	3.220860	3.7943163	4.4663615	5.2533479
35	2.8138624	3.3335904	3.9460880	4.6673478	5.516053
36	2.898278	3.4502001	4.103935	4.8773784	5.7918161
37	2.9852266	3.5710254	4.2680898	5.0908604	6.0814069
38	3.0747834	3.660113	4.4388134	5.3262192	6.3854772
39	3.1670269	3.853717	4.6161659	5.5658990	6.7047511
40	3.2620377	3.9592597	4.8010206	5.8103645	7.0399887

T A B L E II.

The amount of L. 1 annuity, compound interest.

Y.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0300000	2.0350000	2.0500000	2.0500000	2.0500000
3	3.0909000	3.1062250	3.1216000	3.1370500	3.1525000
4	4.1836270	4.2149428	4.2464640	4.2781911	4.3101250
5	5.3091358	5.3624658	5.4163225	5.4707090	5.5256311
6	6.4684098	6.5501521	6.6329754	6.7168916	6.801912
7	7.6624621	7.7794075	7.8982944	8.0191517	8.142008
8	8.8923360	9.0516867	9.2142262	9.3800136	9.549108
9	10.1591061	10.3684958	10.5827953	10.8021144	11.026564
10	11.4638793	11.7313931	12.0061071	12.2887090	12.577892
11	12.8077956	13.1419019	13.4863514	13.841787	14.206787
12	14.1920295	14.6019616	15.0258054	15.4640318	15.917126
13	15.6177904	16.1130303	16.6268376	17.1599132	17.712982
14	17.0863241	17.6769863	18.2919111	18.9321093	19.598631
15	18.5989138	19.2956818	20.0235876	20.7840542	21.578563
16	20.1568813	20.9710297	21.8245311	22.7193367	23.657491
17	21.7615877	22.7050157	23.6975123	24.7417068	25.840366
18	23.4144353	24.4996913	25.6454128	26.8550837	28.132384
19	25.1168684	26.3571805	27.6712294	29.0635624	30.529093
20	26.8703744	28.2796818	29.7780785	31.3714228	33.065954
21	28.6764857	30.2694706	31.9692017	33.7831368	35.719251
22	30.5367803	32.3289021	34.2479697	36.3033779	38.505214
23	32.4528837	34.4604137	36.6178885	38.9370299	41.430475
24	34.4264702	36.6665282	39.0826041	41.6891963	44.501998
25	36.4592643	38.9498566	41.6459083	44.5652101	47.727098
26	38.5530422	41.3131016	44.3117446	47.5706446	51.113453
27	40.7096325	43.7590602	47.0842144	50.7113236	54.669126
28	42.9309225	46.2906273	49.9675830	53.9933332	58.402582
29	45.2188502	48.9107993	52.9662863	57.4230332	62.322711
30	47.5754157	51.6226772	56.0849377	61.0070696	66.438847
31	50.0026781	54.4294709	59.3283352	64.7523877	70.760789
32	52.5027585	57.3345024	62.7014686	68.6662452	75.298829
33	55.0778412	60.3412100	66.2095274	72.7562262	80.063770
34	57.7301765	63.4531524	69.8570085	77.0302564	85.066959
35	60.4620818	66.6740127	73.6522248	81.4966180	90.320307
36	63.2759442	70.0076031	77.5983138	86.1639658	95.836322
37	66.1742225	73.4578694	81.7022464	91.0413442	101.628138
38	69.1594492	77.0288947	85.9703362	96.1382047	107.709545
39	72.2342327	80.7249060	90.4091497	101.4644239	114.095023
40	75.4012597	84.5502777	95.0255157	107.0303230	120.799774

T A B L E III.

The present worth of L. 1 for years, compound interest.

Y.	3per cent.	3½ p. cent.	4per cent.	4½ p. cent.	5per cent.
1	.9708737	.9661835	.9615384	.9569378	.9523809
2	.9425959	.9335017	.9245562	.9157499	.9070294
3	.9151416	.9019427	.8889963	.8762906	.8638376
4	.8884870	.8714422	.8548041	.8385613	.8227024
5	.8626087	.8419731	.8219271	.8024510	.7835261
6	.8374842	.8135006	.7903145	.7678957	.7462154
7	.8130915	.7859909	.7599178	.7348284	.7106813
8	.7894092	.7594115	.7306902	.7031851	.6768393
9	.7664167	.7337309	.7025867	.6729044	.6446089
10	.7440939	.7089188	.6755641	.6439276	.6139132
11	.7224112	.6849457	.6495809	.6161987	.5846792
12	.7013798	.6617883	.6245970	.5896638	.5568374
13	.6809513	.6394041	.6005740	.5642716	.5303213
14	.6611178	.6177817	.5774750	.5399728	.5050679
15	.6418619	.5968906	.5552645	.5167204	.4810171
16	.6231669	.5767059	.5339081	.4944693	.4581115
17	.6050164	.5572037	.5133732	.4731763	.4362966
18	.5873946	.5383611	.493681	.4528003	.4155206
19	.5702800	.5201556	.4746424	.4333017	.3957339
20	.5536757	.5025658	.4563869	.4146428	.3768894
21	.5375492	.4855709	.4388336	.3967874	.3589423
22	.5218925	.4691506	.4219553	.3797008	.3418498
23	.5066917	.4532856	.4057263	.3633501	.3255713
24	.4919337	.4379571	.3901214	.3477034	.3100679
25	.4776055	.4231469	.3751168	.3327306	.2953027
26	.4636947	.4088376	.3600892	.3184024	.2812407
27	.4501890	.3950122	.3468165	.3046913	.2678483
28	.4370767	.3816543	.3334774	.2915706	.2550936
29	.4243463	.3687481	.3206514	.2790150	.2429463
30	.4119867	.3562784	.3083186	.2670000	.2313774
31	.3999871	.3442303	.2964602	.2555024	.2203594
32	.3883370	.3325897	.2850579	.2444999	.2098661
33	.3770262	.3213427	.2740941	.2339712	.2098725
34	.3660449	.3104760	.2635520	.2238958	.1903548
35	.3553834	.2999768	.2524154	.2142544	.1812902
36	.3450324	.2898327	.2436687	.2050281	.1720574
37	.3349820	.2800316	.2342968	.1961992	.1644356
38	.3252261	.2705619	.2252854	.1877504	.1566053
39	.3157535	.2614125	.2166206	.1796654	.1491479
40	.3065568	.2525724	.2082820	.1719287	.1420456

T A B L E IV.

The present worth of L. 1 annuity, compound interest.

Y.	3 per cent.	3½ per cent	4 per cent.	4½ per cent	5 per cent.
1	0.970873	0.966183	0.961538	0.956937	0.952380
2	1.913469	1.899694	1.886094	1.872667	1.859410
3	2.828611	2.801636	2.775091	2.748964	2.723248
4	3.717098	3.673079	3.629895	3.587523	3.545950
5	4.579707	4.515052	4.451822	4.389976	4.329476
6	5.417191	5.328553	5.242136	5.157872	5.075692
7	6.230282	6.114543	6.002054	5.892700	5.786373
8	7.019692	6.873955	6.732744	6.595886	6.463212
9	7.786108	7.607686	7.435331	7.208790	7.107821
10	8.520202	8.316605	8.110800	7.912718	7.721734
11	9.252624	9.001551	8.760476	8.528910	8.306414
12	9.954003	9.663334	9.385073	9.118580	8.863251
13	10.624955	10.302738	9.985647	9.682852	9.393572
14	11.296073	10.920520	10.563122	10.222825	9.898640
15	11.937935	11.517410	11.118187	10.739545	10.379658
16	12.561102	12.094116	11.652295	11.234015	10.837769
17	13.166118	12.651320	12.165668	11.707191	11.274066
18	13.753513	13.189681	12.659296	12.159991	11.689586
19	14.323799	13.709837	13.133939	12.593293	12.085320
20	14.877474	14.212403	13.590326	13.007936	12.462210
21	15.415024	14.697974	14.029159	13.404723	12.821152
22	15.936916	15.167124	14.451115	13.784424	13.163002
23	16.443608	15.620410	14.856841	14.147774	13.488573
24	16.935542	16.058367	15.246963	14.495478	13.798641
25	17.413147	16.481514	15.622079	14.828208	14.093944
26	17.870842	16.890352	15.982679	15.146611	14.375185
27	18.327031	17.285364	16.329587	15.451302	14.643033
28	18.764108	17.667018	16.663063	15.742873	14.898127
29	19.188454	18.035767	16.983714	16.021888	15.141073
30	19.600441	18.392045	17.292033	16.288888	15.372451
31	20.000428	18.736275	17.588493	16.544390	15.592810
32	20.388765	19.068865	17.873551	16.788890	15.802676
33	20.765791	19.390208	18.147045	17.022862	16.002549
34	21.131836	19.700684	18.411197	17.246757	16.192904
35	21.487220	20.000661	18.664613	17.461012	16.374194
36	21.832252	20.290493	18.908281	17.666040	16.546851
37	22.167235	20.570525	19.142578	17.862239	16.711287
38	22.492461	20.841087	19.367864	18.049990	16.867892
39	22.806215	21.102499	19.584484	18.229655	17.017040
40	23.114771	21.355072	19.792773	18.401584	17.159086

T A B L E V.

The annuity which L. 1 will purchase for any number of years to come.

Y.	3 per cent.	3½ per cent.	4 per cent.	4½ p. cent.	5 per cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000
2	.522610	.526400	.530196	.533997	.537804
3	.353530	.356934	.360348	.363773	.367208
4	.269027	.272251	.275490	.278743	.282011
5	.218354	.221481	.224627	.227791	.230974
6	.184597	.187668	.190761	.193878	.197017
7	.160506	.163544	.166609	.169701	.172819
8	.142456	.145476	.148527	.151609	.154721
9	.128433	.131446	.134492	.137574	.140690
10	.117230	.120241	.123290	.126378	.129504
11	.108077	.111091	.114149	.117248	.120388
12	.100462	.103483	.106552	.109666	.112825
13	.094029	.097061	.100143	.103275	.106455
14	.088526	.091570	.094668	.097820	.101023
15	.083766	.086825	.089941	.093113	.096342
16	.079610	.082684	.085820	.089015	.092269
17	.075952	.079043	.082198	.085417	.088699
18	.072708	.075816	.078993	.082236	.085546
19	.069813	.072940	.076138	.079407	.082745
20	.061215	.070361	.073581	.076876	.080242
21	.064871	.068036	.071280	.074600	.077996
22	.062747	.065932	.069198	.072545	.075970
23	.060813	.064018	.067309	.070682	.074136
24	.059047	.062272	.065586	.068987	.072470
25	.057427	.060674	.064011	.067439	.070952
26	.055938	.059205	.062567	.066021	.069564
27	.054564	.057852	.061238	.064719	.068291
28	.053293	.056602	.060012	.063520	.067122
29	.052114	.055445	.058879	.062414	.066045
30	.051019	.054371	.057830	.061391	.065051
31	.049998	.053372	.056855	.060443	.064132
32	.049046	.052441	.055948	.059563	.063280
33	.048156	.051572	.055103	.058744	.062490
34	.047321	.050759	.054314	.057981	.061755
35	.046539	.049998	.053577	.057270	.061071
36	.045803	.049284	.052886	.052886	.060434
37	.045111	.048613	.052239	.052239	.059839
38	.044459	.047982	.051631	.051631	.059284
39	.043843	.047387	.051060	.051060	.058764
40	.043263	.046827	.050523	.050523	.058278

T A B L E VI.

The amount of L. 1 for quarters, compound interest.

Qrs.	3 per cent.	3½ p. cent.	4 per cent.	4½ p. cent.	5 per cent.
1	1.007417	1.008637	1.009853	1.012272	1.012272
2	1.014880	1.017349	1.010804	1.022252	1.024695
3	1.022416	1.026136	1.029852	1.033564	1.037470

APPENDIX.

APPENDIX.

I. Giving a hint, how to resolve some Questions, that were not illustrated in the course of the work.

Chap. III. Rule of Three.

Exam. 40. If a plough goes 2 miles an hour with a furrow of 6 inches, that makes 5280 feet, and
 $5280 : 1 :: 43560 :$

43. Say, $12 : 1 :: 21600$

52. Add 20 to L. 30, is L. 50, or 10000s. the number of people.

67. The hare makes 24 leaps for the hound's 20, and gains another, is 25; and, $4 : 20 :: 20 : 25$ also.

69. You got to the value of L. 35 from Sandy, being equal to both Peter and William, and the price being given, the yards are known.

Chap. VI. Rule of Three in Vulgar Fractions.

Exam. 7. One travels $\frac{1}{2}$, the other $\frac{1}{3}$ miles an hour, reduce to a common denominator, add, and say,
 $\frac{1}{2} : \frac{1}{3} :: \frac{2}{3} : 6\frac{2}{3}$ the time they meet; and then
 $\frac{1}{2} : \frac{2}{3} :: \frac{2}{3} : 22\frac{2}{3}$ miles one has gone.

15. Let the price of the calf be L. 2, the cow L. 7, and the ox L. 21; these added, make L. 30; and
 $\frac{1}{2} : \frac{8}{10} :: \frac{1}{2}$ the value of the calf, &c.

16. Say, $\frac{1}{2} : \frac{4}{9} :: \frac{2}{3} : L. 12$ 10 the gain; half whereof is the gain on the fine; and divided by 100, gives a yard of the coarse.

18. Add

18. Add all the fractions, $\frac{1}{8}$, $\frac{1}{10}$, &c. subtract their sum from 1; make the remainder your first number; add all the folios missed 225, for your second, and your third number is 1.

19. B will do $\frac{1}{3}$ in 30 days; wherefore, in the 24 days he wrought already, he has done $\frac{8}{10}$; to this add $\frac{1}{5}$ done by C, and $\frac{1}{3}$ not yet done, makes $\frac{8}{10}$; take this from 1, remains $\frac{2}{10}$, which must be the 12 acres done by A: Then say, $\frac{2}{10} : \frac{1}{3} :: 1 : 60$ acres, the whole field.

Chap. VIII. *Rule of Three in Decimals.*

Exam. 20. Divide 25 by 8, 11 by 4, and 5 by 2, and with the sum of the quotients divide 67, gives the answer; for this divisor would be what all the 30 would do in a day.

24. When he would have L. 100 in one case, and want L. 100 in the other, L. 200 must be a shilling for every pound of his debt.

25. Add together $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{10}$, or these decimals, .2, .1, .05; subtract their sum from 1, and with the remainder divide 15600.

§ 2. *Partnership.*

Exam. 4. Add $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, or .33, .25, .2; subtract their sum from 1, and with the remainder divide L. 52, gives the whole gain, which divide, as in the question; multiply D's stock and time, and say, 52 : 1040 :: 80 : 1600 A's stock and time; 52 : 1040 :: 60 : 1200 for B; which divided by 600 his stock, gives his time, &c.

Chap. XI. *Tare and Tret.*

Exam. 7. Say, 100 : 4 :: 16.5 : .66 the tret, &c.

9. Say,

9. Say, $100 : 4 :: 5000 : 200$ the tret; so add the double of it to the neat.

14. Divide L. 75 by 7s. 6d., gives the lb. neat, and so of the rest.

15. Suppose you bought 100lb. say, $9 : 100 :: 110$ work Inverse, you have $81\frac{1}{9}$; so you must pay but for $\frac{1}{9}$ of the gross, to gain 10 per cent. so $\frac{1}{9}$ must be allowed for tare and tret; if it held out in the sale, you would gain 16.6 per cent. therefore $116.6 : 100 :: 90 : 77\frac{1}{2}$; it holds out in the sale $\frac{27}{32}$ of the gross.

§ 2. *Loss and Gain.*

Exam. 22. Say, $110 : 100 :: 2.6 : 2.42$ s.

23. Say, $100 : 90 :: 9 : 8.1$

24. You gain L. 60 on your hundred, at 12s. each beast, so will his gain be L. 40; and as he gained 10 per cent. say, $10 : 100 :: 40 : 400$ his outlay; the rest is easy.

26. The amount of 250 yards, at 15s. 4d. is L. 191.6, and 12 per cent. gain makes it L. 214.6; from this take L. 100, remains L. 114.6; then say, $.8 : 1 :: 114.6 : 143.3$; from this take 125, viz. half 250, there remains 18.3, half the small piece.

27. He gained 20 per cent. is L. 6.25 gain, also he gained the price of the hog on the ox, and the price of the wedder on the hog, and nothing on the wedder; therefore the price of the hog and wedder must be his whole gain: subtract then 6.25 the gain, from 31.25 the outlay, there remains L. 25 the price of the ox, and 20 per cent. gain on this, or L. 5 is the price of the hog, &c.

30. Find the present worth of 31s. 6d. which is 31.11s. and then say, $17 : 31.11 :: 16 : 29.281$; subtract this from 30.

Chap.

Chap. XII. *Barter.*

Exam. 7. The value of 5 yards shalloon; and 2 of buckram, is 9s. 6d. therefore say, by the Inverse rule, $19 : 100 :: 95$ the answer is 200; which, doubled, is the yards buckram, and its product, by 5, is the yards of shalloon.

8. The real value of A's cheese is L. 30, the value in barter L. 31 10; then say, $7 : 1 :: 630 : 90$ stones; the real value of which is L. 29 5; take this from L. 30.

§ 2. *Factorage.*

Exam. 5. Say, $105 : 100 :: 1146.6 : 1092$ and $104 : 100 :: 1092 : 1050$ to be remitted:

6. The amount of the cloth is L. 5512; from which deduct commission on sales, $106 : 100 :: 5512 : 5200$; and on returns, $104 : 100 :: 5200 : 5000$; one-half of which divided by 2, gives the sugar; and multiplied by 6, gives the rum; add L. 1000 to the neat proceeds, half of this is L. 3000, which divide by the cwt. sugar, also by the gallons rum, gives their selling prices.

§ 3. *Insurance.*

Exam. 10. Say, $98 : 100 :: 240.1 : 245$ and $245 : 22.05 :: 100 : 9$

Chap. XIV. *Rule of False.*

Exam. 9. Suppose 1lb. cinnamon 12s. 1lb. pepper must be 7s. 4d. at these prices the second amount would be L. 60.3, so the error is 1.

10. Suppose 3 lb. of gold, and 1 of silver; the gold fills $\frac{3}{4}$ of the mould, and the silver $\frac{1}{4}$, that is, $\frac{1}{3}$, so the error is $\frac{1}{12}$ too little; if you suppose 2 of each, it will be $\frac{1}{12}$ too much.

15. Suppose the lbs. fold, three times that number will be those unfold; divide the sum of both, that is, all

all the lbs. by $\frac{1}{12}$ part of the unfold, and what the quotient differs from $\frac{1}{3}$ of those sold, is your error.

Chap. XVIII. *Practical Questions.*

Exam. 1. The present worth of L. 292, due 100 days hence, discounted at 5 per cent. is L. 288.05, and 150 of principal may be found to amount to this in the time mentioned.

3. The difference of one year's interest of the two bills is 12s. and this dividing L. 5 2, quotes 8 years, 6 months; which subtract from the given year.

II. QUESTIONS AT RANDOM.

Quest. 1. A gentleman's estate, heritable and movable, worth L. 1500, consisted of 2 farms, 4 yoke of oxen, 8 horses, 16 hogs, 32 cows, and 64 sheep; being pushed for money, he sells his 2 farms, and 2 of every other article, and then his remaining subject was L. 415; he sold again 2 of each remaining article, and what was left was worth L. 330; next he sold 4 off every article, and there remained to the value of L. 240; again sold 8 of every article, and there was left L. 156; again he sold 16 of each remaining article, and there was left L. 40 value; lastly, he sold 32 of the only article that remained, and no more was behind: I demand the value of a farm, a yoke of oxen, a horse, a hog, a cow, and a sheep?

Answer, A farm L. 500, a yoke of oxen L. 20, a horse L. 12, a hog L. 3.25, a cow L. 6, and a sheep L. 1.25.

2. A man distributing apples among 6 ladies; gives the first half his number, and half an apple over; to the second half the remainder, and half one over, and so on to the last, who got all he had left, viz. 40: Required the number of apples? *Answer,* 1311.

3. Bought

3. Bought 6 pieces broad cloth, and paid in hand the price of 100 yards; bought again 6 pieces more, paid half in hand, and half my former balance, and then owed L. 36 16 8; bought a third time 6 pieces, and paid L. 60, and now I owe L. 27 16 8: I demand the yards in a piece? Answer, 30.

4. With $\frac{1}{3}$ of my money bought 60 casks of brandy, and 40 of rum; with another third, 50 of brandy, and 48 of rum; with my last third, 30 of brandy, and 64 of rum, always at the same price; and find I have paid L. 200 more for rum than for brandy: What was my money? Answer, L. 1320.

5. Paid a guinea for a basket of lemons and oranges, paying 2d. for every orange, and 1d. for every lemon; and selling all at $1\frac{1}{2}$ d. a-piece, I gained 1s. How many were of every kind?

Answer, 100 lemons, 76 oranges.

6. Bought 120 wedders and ewes for L. 60, gave 5s. more for a wedder than a ewe; in the sale gained 1s. on each ewe, but lost a shilling on each wedder, and gained L. 3 clear on the whole: I demand what cost a wedder and a ewe?

Answer, A wedder 13s. 9d. a ewe 8s. 9d.

7. Put out a sum of money to interest, at 5 per cent. and when it had continued 5 years, received of interest and part of the principal, L. 200; let the remainder continue 5 years longer, and then again received L. 200; and letting the remainder continue 5 years more, received in full of principal and interest, L. 200: I demand the sum put out at first?

Answer, L. 390 8.

8. Bartered 12 pieces scarlet for 19 pieces blue, and find I am L. 10 a gainer; again bartered 9 pieces of the same scarlet for 14 pieces blue, and gained L. 5: What was the value of a piece of each?

Answer, Scarlet L. 15, blue L. 10.

9. A man leaves L. 10876 13 4 among his three sons, in such sort, that the first putting out his part
to

to interest, at 3 per cent. the second his, at 4 per cent. and the third his, at 5 per cent, in 8 years time the three amounts came to be equal: I demand each of their shares of the legacy?

Answer, L. 3850, L. 3616.6, and L. 3410.

10. Having bought and sold 6 bales of goods, I gained L. 12; but buying 6 more, 5 per cent. dearer, and selling them, 5 per cent. cheaper than before, I lost L. 12: What cost a bale of goods at first?

Answer, L. 39.

11. A piece of toll-road is made by 8 men in 10 days, 400 falls long; and again 6 of these men, in 8 days, make 220 falls of another road, 1 foot broader than the former: The breadth of each road is required? Answer, L. 11 and 12 feet.

12. A company of 20 men, in drinking 30 bottles of wine, have $10\frac{1}{2}$ glasses a-piece; again, 12 men drinking 18 bottles, that hold each a gill more than the former, have 12 glasses each: Required the capacity of the bottle? Answer, 7 and 8 gills.

13. Three women carry to market each an equal number of eggs; the first sells 3 for a penny, the second 5 for 2d. the third 7 for 3d. Now, the last got 10d. more than the first, and 3d. more than the second, how much money got each?

Answer, The first 35d. the second 42d. the third 45d.

14. There are 5 droves of cattle, 999 in all, divided so, that there are 17 more in the second than in the first, the third drove is double the first, the fourth equal to both the second and third, and the fifth equal to the third and fourth: How many are in each drove? Answ. 79, 96, 158, 254, 412.

15. There are 100 geese, hens and turkeys, value L. 10 geese at 3s. hens at 1s. and turkeys at 2s. 6d. each: How many are there of every kind?

Answ. Geese 2, 5, 7, &c. to 47. hens 34, 35, &c. to 49. and turkeys 64, 60, 56, &c. to 4.

E e

16. Bought

16. Bought 50 hogheads of porter, and 60 of beer, for L. 380, sold all again for L. 408, having gained 10 per cent. on the beer, and 5 per cent. on the porter: Required what cost a hoghead of each?

Answer, Beer L. 3, porter L. 4.

17. Two men set off from one place; the first goes east, the second south-east at first; but by the time he is distant 20 miles from the east and west road, he turns towards the north-east quarter, and at last falls in again with the first man: How many miles has he now travelled? Answer, 56.56 miles.

18. Two ploughs labour 2 acres of land in a day; the first goes 184 feet in a minute, with a furrow 7 inches broad, the other 196 feet, with a furrow 6 inches broad: I demand how much is laboured by each?

Answ. The first 1 acre, $7\frac{1}{4}$ falls, the other 3 roods, $32\frac{1}{4}$ falls.

19. There is a square piece of land, a side of which is 1 mile; a man has to go round this with a plough till he labour 150 acres: How many rounds will this take? Answer, 660.

20. I have L. 100 in guineas, crowns and shillings, 1000 in all: How many of each?

Answers to this are many.

21. Bought a quantity of candles, at 1d. a-piece; and again the same quantity, at 8d. the pound; the last was a crown cheaper, and selling the whole at 10d. a-lb. gained only 20d. I demand the quantity?

Answer, 20lb.

22. A gentleman, who had a number of labourers, paid them L. 3 12 every day; but afterwards hiring 6 more, and lessening their wages 2d. each, he has them for the same money: What was the number of labourers? Answer, 48 at first.

23. How long will it be ere the Gregorian, or new style, gain a year of the Julian, or old style?

Answer, 48666.6 years.

24. A

24. A teacher, who has 60 scholars, receives L. 3 5 a-week, at the rate of 8d. from every reader, 12d. from a writer, and 18d. from a figurer: Required the number of each? Answers are many.

25. Divide 100 into 4 proportional numbers in triple ratio. Answer, 2.5, 7.5, 22.5, 67.5.

26. If a clock gains 15 minutes in 24 hours, what is the length of her pendulum? Answ. 38.39 inches.

Rule. A pendulum that vibrates 60 times per minute, is 39.2 inches long, this is called the *Standard*; for any other number of vibrations, say, As the square of the number of vibrations, to the square of 60, so the standard to the length.

27. How long must a pendulum be, to vibrate only twice in a minute? Answer, 980 yards.

28. How oft will a pendulum, of a mile long, vibrate in a minute? Answer, $1\frac{1}{2}$ nearly.

Say, As the given length to the standard, so the square of 60, to the square of the number sought.

29. Two travellers set out from one place, and after they have travelled 4 days, the one is 10 miles before the other; but slackening his pace $\frac{1}{6}$ part, in 4 days more the other makes up with him: How many miles did each travel a-day? Answer, 30, and 27.5.

30. A man, who would proportion his expences to his income, spends so much per month, but at the year's end has two months expences over; next year he increases his expences L. 5 per month, and by the end of 11 months, has done, both with his year's income, and the last year's overplus: Required his income? Answer, L. 154.

31. Bought an estate of 700 acres, at 25 years purchase, paid one half in hand, and gave bill for the other, at 5 per cent. payable in 6 months; after 2 years sold one half of this estate, also at 25 years purchase, but the rent was then raised $\frac{1}{5}$, and this clearing

ing off my bill to the former proprietor, and L. 54 13 9 over: What was the rent per acre?

Answer, 5s. at first.

32. Bought 3 houses, whose rents are in proportion as 2, 5 and 7, and paid for the first 10 years purchase, for the second 12, and for the third 15, and so gave L. 740 for all three: What is the rent of each house?

Answer, L. 8, L. 20, and L. 28.

33. Bought cloth, at 12s. 6d. the yard, and got a yard to the score; sold it again, at 13s. 4d. and gave a yard to the dozen, and thereby have gained L. 100: How many yards had I? Answer, 4963 $\frac{1}{4}$.

34. Three families spend as follows: The first L. 33 in 5 weeks, the second L. 52 13 in 9 weeks, the third L. 83 1 in 11 weeks: How must L. 1000 be shared among them, so as all three may be supported an equal time?

Answer, First L. 330, next L. 292.5 and L. 377.5.

35. Two merchants enter into company; A puts in L. 50 at first, and L. 10 every month after; B puts in L. 147, which continues to the end of the time, when A's gain was to B's as 5 to 7: I demand the number of months? Answer, 12 months.

36. A farm is to be sold at 20 years purchase ready money, or at 25 years purchase, and 5 years credit: Which offer is the best? Answer, Equal.

37. Two men depart from one place; the one goes 80 miles east, and then 32 miles north; the other goes 40 miles south, and then 26 miles east: How far are they now asunder? Answer, 90 miles.

38. Hired to-day 60 men, reapers, for 50s. and 40 women for 30s. to-morrow I intend to keep only 50 of both, in such proportion, that their wages may be 40s. How many men and women must I keep?

Answer, 30 men, 20 women.

39. A family that had L. 1000 at interest, at 5 per cent. expended it all, principal and interest, in 10 years:

years: What was their rate of living per annum?

Answer, L. 129 10 nearly.

40. Three men, A, B and C, put in L. 1000 together into stock, A to be manager, and to have $\frac{1}{10}$ of the gain of the other two for his pains; after 8 months C withdraws, and receives L. 36 of gain; in 4 months more A and B divide their gain, and have L. 45 of gain each, exclusive of what A got from C, but his commission from B included: I demand the stock of each?

Answer, A, L. 266.6, B, L. 333.3, C, L. 400.

41. A man distributes $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his subject among his four sons, the remainder he sends to sea, receives 20 per cent. increase from abroad, and again distributes of this $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$, as before, and now has L. 1000 left: It is required what was his subject at first? Answer, L. 333333 6 8.

42. In selling cloth, at 5s. 4d. the yard, I gained L. 7 on the pack, and raising my price to 6s. I gained 20 per cent. How many yards in a pack?

Answer, 420.

43. Bought 60 cwt. of tea, 70 of tobacco, and 80 of sugar, for L. 2980; a cwt. of tobacco was L. 10 dearer than a cwt. of sugar, and the cwt. of tea L. 10 dearer than both together: I demand what cost the cwt. of each?

Answer, Sugar L. 4, tobacco L. 14, tea L. 28.

44. Bought 20 bales of goods, at 9 months credit; which sold for ready money, at 5 per cent. profit, and putting the money to interest for 9 months, at 5 per cent. the amount cleared me L. 100 over the price of my goods: Required the price of a bale?

Answer, L. 55 18 $10\frac{1}{2}$ nearly.

45. Paid L. 18 5 for limestone, at 22d. per ton, and coals, at 18d. the cart: I demand the quantity of each, there being 2 carts of the latter, for 5 tons of the former? Answer, 150 tons, 60 carts.

46. In

46. In a fleet of war ships there are 3180 guns, and 24000 men; a line-of-battle ship having 90 guns, and 700 men, and a frigate 40 guns, and 250 men: I demand the number of each?

Answer, 30 ships of the line, 12 frigates.

47. Planting a number of trees in form of a square, I had 96 over after compleating the square; but increasing every row by 5, I wanted 949 to complete the square: Required the number of trees?

Answer, 10500 trees.

48. A labourer expends every day a third of his wages on his board, and at every week's end one-third of his saving at the tavern, and at the year's end a third of his year's saving upon clothes, and still has as much left as comes to 3d. a-day through the year, calling it 312 working days: I demand his wages per day?

Answer, $10\frac{1}{8}$ d.

49. A man works a certain number of days, for 32d. a-day; again, a certain time, at 2s. and lasting some time at 20d. his board all along was 6d. a-day, and he has saved L. 7 16 8: I demand how many days he wrought at the several wages, having been 100 days in all? Answers are various here.

50. A shepherd, in numbering his sheep and goats, finds he has 7 sheep for 2 goats; and, afterwards selling off 1200 sheep and 500 goats, he then has 16 sheep for 3 goats: How many of each had he at first?

Answer, 2800 sheep, 800 goats.

51. Remitted to Ireland L. 500, at 8 per cent. and at the same time L. 600 to London; after some time my Irish correspondent drew upon my London one, at 10 per cent. and sent me home both remittances together, at $9\frac{8}{9}$ per cent. and L. 20 less than they were sent away: What had I for my L. 100 at London?

Answer, L. 98.

52. Remitted L. 1000, one part to France, at 30d. per crown, and thence to Holland, at $4\frac{1}{2}$ crowns per pound Flemish; the other part to Holland, at 35s. Flemish

Flemish per pound Sterling, and thence to France, at $4\frac{1}{2}$ crowns per pound Flemish; and then drew upon France, at 32d. per crown, and upon Holland, at 36s. per pound Sterling, and have gained L. 3.55: Required the remittance to each place?

Answer, L. 600 to France, L. 400 to Holland.

53. Bought an estate, at 20 years purchase, for ready money; sold it again, at 25 years purchase, got half in hand, and gave 12 years credit for the other, without interest; by the end of which time I will be L. 100 a gainer, or discounting just now, I am L. 62 10 a-gainer: Required the rent of the estate?

Answer, L. 200.

54. A gentleman expendeth every day $\frac{1}{1000}$ part of his income; also every week $\frac{1}{10}$ part additional; and every month $\frac{1}{8}$ part more; and at the year's end has L. 123 saved: Required his income?

Answer, L. 1800.

55. A general, who had 20 footmen for each horseman, loses in a battle 10000 foot and 700 horse, and now has 30 foot for every horse: How many had he at first? Answer, 22000 foot, 1100 horse.

56. Divide 20 into two parts, so that the one part may be 20 times as great as the other.

Answer, 19.047, and .952.

57. Divide 1414 guineas among 7 men, in Arithmetical Proportion, so as the first may have 100 times as many as the last. Answer, 4, 70, 136, &c. to 400.

58. Borrowed L. 234, at 5 per cent., and at the same time L. 500, at 4 per cent.; after some time, going to clear off, I find it will require L. 760 8 4 to discharge both: I demand how long I have had this money? Answer, 10 months.

59. Borrowed L. 220, at 5 per cent. and again 12 months after L. 340, at 4 per cent. and now going to clear off these debts, I find it will take L. 607 18: I demand the times I have kept these sums?

Answer, The first 30 months, the second 18.
60. Paid

60. Paid L. 1000 for a cargo of goods neat cost and freight home ; in the sale I gained a fifth of the prime cost, provided I had got ready money ; but, giving a year's credit, the interest for that time, at 5 per cent. is a fourth part of the freight : Required the cost, freight and gain ?

Answer, Cost L. 769 $\frac{1}{11}$, freight L. 230 $\frac{1}{13}$.

61. Insured on board the Nereid L. 1000 value, at 15 per cent. and 2 per cent. discount in case of loss : the premium passes to account, and so it happens, that part of the cargo being lost, I received from the underwriters in full L. 400 : I demand the value lost ?

Answer, L. 561.224.

62. A man intends to stay as long in a place as his money will support him, at 5s. per week ; but so it happens, that he spends the first week 6s. the next 7, the next 8, &c. till all is spent, and he is obliged to leave the place 11 weeks sooner than he proposed at first : I demand what money he had ?

Answer, 5 guineas.

63. A ship's hull and cargo are valued at L. 2800 ; the owner has the ship insured, at 5 per cent. and the merchant the cargo, at 8 per cent. The premiums being passed to account, and the ship lost, the underwriter pays them L. 2571, being allowed 2 per cent. discount : What will each receive ?

Answer, The owner L. 1581, the merchant L. 990.

64. On a ship's load of tobacco and sugar, allowed $\frac{1}{16}$ part of the sugar, and $\frac{1}{16}$ part of the tobacco as tare, and tret on the sugar only ; selling the tobacco at L. 5 per cwt. and the sugar at 40s. neat ; received L. 57300 : Required the gross weight of each, the whole being 868 tons ? Answ. 400 tons tobacco, 468 tons sugar.

65. The number of the month, wherein I was born, is to the day of the month, as 4 to 9, and the sum of both to their product, as 13 to 108 ; moreover, the square of their sum added to the square of their difference,

ference, abating their difference, gives the year of our Lord: Required the time of my birth?

Answer, 27th December, 1731.

66. Let a sum of money, at 5 per cent. all in guineas, and after 5 years received principal and interest, all in crowns, and find the sum of the pieces, viz. the guineas lent, and crowns received, to be 1800: Required the sum lent? Answer, 288 guineas.

67. A man leaves his subject to his sons, as under: To the eldest $\frac{1}{7}$ and L. 100 more, to the second $\frac{1}{7}$ of what was left and L. 200 more, to the third $\frac{1}{7}$ the remainder, and L. 300 more, and so on with the rest, and finds all have shared alike, and his subject exhausted among them: Required his subject?

Answer, L. 4200.

68. Being promised change for what gold I had, I threw down a guinea, and received a shilling; being bid throw down another, I had for that 2s. and for the next 3s. and increasing thus to my last guinea, I have my full change: Required my number of guineas? Answer, 41.

69. Selling 12 cwt. of sugar, and 20 cwt. of tobacco for L. 322, I gained $16\frac{2}{3}$ per cent. and again selling 20 cwt. of sugar, and 12 cwt. of tobacco for L. 221, I gained $8\frac{1}{3}$ per cent. What cost an cwt. of each? Answer, Sugar L. 3, tobacco L. 12.

70. Divide 100 into four such parts, that if you add 4 to the first part, or subtract it from the second, or if you multiply the third, or divide the fourth still by 4, the sum, remainder, product, and quotient may be always the same number. Answer, 12, 20, 4 and 64.

71. Betwixt two parallel lines, 60 inches asunder, are laid two rows of guineas, crossing each other in form of the letter X, and whose ends are 25 inches from each other: Required the number of guineas?

Answer, 129.

72. Two merchants bought in company 60 tons iron, and 20 of steel; in the sale, A receives payment

F f

of

of 5 tons steel, and B of 15, both selling at 10 per cent. profit, and then B pays A, L. 137 10, to even accounts; of the iron, A receives payment of 35 tons, and B of 25, and A pays B, L. 110 to even accounts: What cost a ton of each? Answer, Iron L. 20, steel L. 25.

73. Four men put in L. 3892 in company together, A's money for 4 months, B's for 5, C's for 6, and D's for 7; and for every penny A takes of the gain, B takes a shilling, C a crown, and D a guinea: I demand the stock of each?

Answ. A, L. 20, B, L. 192, C, L. 800, and D, L. 2880.

74. A man bequeaths L. 5220 to his four sons, in such sort, that the second has $\frac{1}{7}$ the share of the first, the third $\frac{6}{7}$ of the second, and the fourth $\frac{5}{6}$ of the third: Required the share of each?

Answ. L. 1980, L. 1260, L. 1080, and L. 900.

75. Bought a quantity of timber, viz. wainscot and mahogany; the wainscot at 3s. the mahogany at 5s. the foot; the amount come to L. 800; and selling the mahogany at 6s. and the wainscot at 4s. I gained $22\frac{1}{2}$ per cent. on the whole: Required the quantities?

Answ. Mahogany 2600 feet, wainscot 1000 feet.

76. The product of two numbers, divided by their sum, quotes 30, and divided by their difference, quotes 120: Required the numbers?

Answer, 48 and 80.

77. A jockey buys two fine horses; the first L. 10 dearest; he gains on the first 10 per cent. on the other 12; yet receives L. 5 more for the first: Required the prices? Answer, L. 310, and L. 300.

78. A and B put in money together; A for 8 months, and B for 6; A's gain came to be double to B's; afterwards each increased their stocks L. 100, and continuing in trade the same time as before, A's gain was $1\frac{1}{2}$ times B's: Required their stocks at first?

Answer, A's L. 50, B's L. 33.3.

79. Three

79. Three men talk of their ages thus: Says, A to B and C, I am as old as you both; says B to C, I am six times your age; and says C, my number of years is the square root of the sum of all our years: Required the age of each?

Answer, A 90, B 84, C 14.

Rule, Suppose any three numbers you please, in the proportion given, and divide their sum by the least number, you have the age of the youngest.

80. There are three numbers; the sum of the squares of the first and second is 520; of the first and third 680, and of the second and third 808; Required the numbers? Answer, 14, 18, and 22.

81. Lent L. 100 on these terms, viz. that I was to be paid back L. 1 at the first year's end, 2 at the second, 3 at the third, &c. till I be cleared: How long time will this take, accounting simple interest at 5 per cent.? Answer, 18 years, 84 days.

A L G E B R A.

I N T R O D U C T I O N.

THE marks explained at the beginning of the Arithmetical part, are also used in Algebra, where $+$ is called the *positive*, and $-$ the *negative sign*; there is also this mark $\sqrt{}$, which signifies the extraction of a root; as, $\sqrt{a}$, signifies the square root of a ; and $\sqrt[3]{a}$, the cube root of it; a line, or dash, over two or more quantities, shews them to be taken as one compound quantity; thus $\overline{a+b-c}$ denotes the remainder, when c is subtracted from the sum of a and b . A number prefixed to any quantity, multiplies that quantity, and is called its *co-efficient*; as, $3a$ is three times a ; and when no figure is set down, *unity* is to be understood. The square, cube, or biquadrate of a , or other quantity, may be wrote thus, aa , aaa , $aaaa$, or, for shortness, rather thus, a^2 , a^3 , a^4 , &c. When a positive quantity stands first in a series, its sign is omitted, but a negative sign must never be omitted. The product of two, or more quantities, is expressed by the letters set together; thus the product of a and b , multiplied together, stands thus, ab ; of a , b and c , thus, abc , or cba , for it matters not which be first.

Addition of Algebraic Quantities.

If quantities are of the same kind, or expressed by the same letter, add their co-efficients, and set their sum before the letter, or quantity, with the same sign; thus $3a$ and $2a$ make $5a$, and b added to $6b$, is $7b$;
also,

also, if they are negative quantities, $-9a$ and $-12a$ make $-21a$. See more examples.

$$\begin{array}{r}
 a + b - c \\
 10a + b - 2c \\
 \hline
 11a + 2b - 3c
 \end{array}
 \qquad
 \begin{array}{r}
 4a - b + c \\
 5a - 2b + 8c \\
 7a - 6b + 7c \\
 \hline
 16a - 9b + 16c
 \end{array}$$

But if the quantities have different signs, subtract the lesser co-efficient from the greater, prefix the remainder to the quantity, with the sign of the greater co-efficient, thus; add $-9a$ to $12a$, the sum is $3a$; but $-12a$ to $9a$, the sum is $-3a$; for the negative quantity, being less than nothing, diminishes, instead of increasing a quantity: So a and $-a$ added together, make 0 .

Examples.

$$\begin{array}{r}
 6a + 12b - 7c \\
 -9a - 3b + 4c \\
 \hline
 -3a + 9b - 3c
 \end{array}
 \qquad
 \begin{array}{r}
 a + b + c \\
 5a - b - c \\
 2a + 5b + 7c \\
 \hline
 8a + 5b + 7c
 \end{array}$$

When many quantities, both positive and negative, are given, the easiest way is, to collect the positives into one sum, and the negatives into another, and then subtract their co-efficients, as directed.

Heterogeneous quantities, or such as are expressed by letters, are added by setting the mark of Addition betwixt them; thus $a+b$ denotes the sum of a and b and $a+b+c$ is the sum of these three quantities.

Subtraction

Subtraction of Algebraic Quantities.

When quantities are homogeneous, and have the same signs, subtract the one co-efficient from the other, setting the same sign before the remainder; so $6a$ from $10a$, leaves $4a$; or, $-8a$ from $-11a$, leaves $-3a$. If a greater quantity is to be taken from a less, subtract the lesser from the greater, but give the remainder a contrary sign; thus $10a$ from $4a$ rests $-6a$; or, $-10a$ from $-4a$, rests $6a$.

If quantities have unlike signs, add their co-efficients together, and to their sum prefix the sign of the minuend; thus, $6a$ from $-4a$, rests $-10a$; or, $-4a$ from $6a$, rests $10a$. See examples of all these.

$$\begin{array}{r} 6a+4b-8c \\ 2a+3b-6c \\ \hline 4a+b-2c \end{array}$$

$$\begin{array}{r} 8a+2b-3c \\ 6a+9b-5c \\ \hline 2a-7b+2c \end{array}$$

$$\begin{array}{r} 6a-b+c \\ -2a+2b-5c \\ \hline 8a-3b+6c \end{array}$$

Heterogeneous quantities are subtracted, by putting the sign of Subtraction betwixt them, thus: If b were to be subtracted from a , the remainder is $a-b$; or, a from b , remains $b-a$. If compound heterogeneous quantities are to be taken from one another, change the sign of every part, or member of the subtrahend, and set it after the minuend, thus: If you subtract $a+b-c$ from $d+e$, there remains $d+e-a-b+c$.

Multiplication of Algebraic Quantities.

If the factors have both the same sign, whether positive or negative, the product is positive; but if they have different signs, it is negative, thus: a multiplied by b , or $-a$ by $-b$, give ab a positive product; but a by $-b$, or b by $-a$, give $-ab$ a negative one. In compound

compound quantities, multiply every member in the one factor by every member in the other, observing carefully the signs, and setting quantities of the same kind below one another, add their sums together, as taught in Addition. See examples.

Multiply $a+e$ By $b+d$ Product $ab+be+ad+de$	$a+b-c$ $a+b$ $aa+ab-ac+bb-bc$ $+ab$ $aa+2ab-ac+bb-bc$	$a+e$ $a+e$ a^2+ae+e^2 $+ae$ $a^2+2ae+e^2$
$a-e$ $a-e$ a^2-ae+e^2 $-ae$ $a^2-2ae+e^2$	$a+e-y$ $a-e+y$ $a^2+ae-ay-e^2+ey-y^2$ $-ae+ay$ $+ey$ a^2 0 0 $-e^2+2ey-y^2$	

Division of Algebraic Quantities.

Divide the co-efficient of the dividend by that of the divisor, when possible, and putting that figure in the quotient, set all the letters of the dividend after it that are not in the divisor, and for the sign, observe the same rule as in Multiplication. This $6abc$, divided by $2a$, quotes $3bc$, or by $-2a$, quotes $-3bc$. When any one letter is found in every member of a train of quantities, they are divided by that letter, when it is struck out of them all thus, $ab+ac-ad$, being divided by a , the quotient is $b+c-d$. When the quantities are of different kinds, or the co-efficients incommensurable, write the divisor under the dividend, in form of a vulgar fraction, so if a be divided by b , the quotient

quotient is $\frac{a}{b}$, or $\overline{ab+cd}$ by $\overline{a+b}$, the quotient is $\frac{ab+cd}{a+b}$. Such division is what will be most used in the following work.

Extraction of Roots.

The root of any simple power is known by inspection; thus the square root of aa , or a^2 , is a ; the cube root of a^3 also a ; and to any compound quantity prefix the radical sign; thus the square root of $\overline{a+b}$ is $\sqrt{a+b}$, and $\sqrt[3]{a+b}$ the cube root of it.

It may be observed, that fractions are every way managed in Algebra, as in Arithmetic, respect being had to the rules of Addition, Subtraction, &c. above delivered.

Of Equations.

By considering the nature of any question, that can be proposed, we will always find something in it, given or required, equal to some other thing given or required; and setting these opposite to one another, with $=$, the *sign of equality*, betwixt them, we are said to raise an equation: Thus, if a man say, He had lost L. 20, and had still 40 left; to know what he had at first, calling his money a , this equation arises, $a - 20 = 40$: Or, if one say, Had he L. 40 more, he should have L. 100; this is the equation $a + 40 = 100$: If he said, His money divided among his 6 sons, would be L. 10 to each of them, then $\frac{a}{6} = 10$: Or if it were demanded, How many horses would I buy for L. 60, at L. 10 each? consider the number of horses, multiplied by

G g

the

the price of one, would be L.60; so you have this equation, $10a=60$, a being the number of horses.

For the reduction of these equations, see next paragraph; meantime we may observe, That as many equations must always arise from a question, as there are quantities required, and no more; otherwise, the question will be doubtful or impossible. Unknown quantities, or such as required, are represented by the Italic small vowels, or by the last letters of the alphabet, x , y , z , by some; however, this is matter of choice. Given quantities are represented by absolute numbers, as they are given, if you please, and which is perhaps the plainest way for beginners; but it is much more concise and commodious, to represent them also by letters.

Reduction of Equations.

After an equation is raised, we are next to reduce it; that is, to bring the unknown quantity to stand alone on one side of the equation, and known quantities on the other, so that the required quantity may also be known; and to do which, mind the following observations.

Any member may be removed to a different side of the equation, by changing its sign, this is called *Transposition*; for if a member on one side of an equation be negative, and you place it on the other side positive, you really add it to both sides; and when equal quantities are added to equal quantities, the sums must be equal: So in the first equation, last paragraph, $a-20=40$; if you add 20 to both sides, you have $a=60$. Or if a member be positive on one side, and you remove it to the other, with a negative sign, this subtracts it from both sides, for equal quantities, taken from equal quantities, give equal remainders: So in the second equation above, $a+40=100$;
if

if you subtract 40 from both sides, you have $a=60$, so a is also known.

Again, If one side of an equation is divided by any number or quantity, omit the division on that side, and multiply the other side by the divisor, and the sides of the equation will still be equal; for they are both multiplied by the same thing; and equal quantities, multiplied by equal quantities, give equal products;

in our third equation above $\frac{a}{6}=10$; mul-

tiple both sides by 6, and you have $a=60$: Or if one side is multiplied by any quantity, drop that quantity on one side, and divide by it on the other, and you have both sides divided by it, so the sides of the equation will always be equal; as in the fourth equation above, $10a=60$; divide both sides by 10, and you have $a=6$.

In more complex cases we proceed step by step, using one of these rules after another, till we attain our purpose: When any quantity is found on both sides of an equation, it may be struck out of both, whether positive or negative, or equal quantities, on the same side of an equation, with different signs, destroy one another, and so both may be dropt; sometimes it may be advantageous, to subtract one equation from another, or add them together, as circumstances may require; but this will be better understood afterwards, when the learner has seen something of the practice of this curious science, which, however abstruse and subtle in its operations, we see it has its dependence on very simple, and self-evident principles.

I. OF SIMPLE EQUATIONS.

Prob. 1. What number is that, from which, if you subtract 4, multiply the remainder by 5, add 12 to the product, and divide the sum by 7, the quotient may be 21? Answer, 31.

For

For the number put a ; subtract 4, you have $a-4$; multiply by 5, you have $5a-20$; add 12, you have $5a-8$; which, divided by 7, quotes 21: So your equation is $\frac{5a-8}{7} = 21$. To reduce which by the rules formerly given, multiply by 7, you have $5a-8=147$; add 8 to both sides, you have $5a=155$; divide by 5, you have $a=31$. The work would stand, as in the next problem.

Prob. 2. In my first 10 years of trade, tripled my stock; but in the next 10, lost L. 1200; in the last 10, doubled what stock I had; and now, dividing my subject among my 6 sons, each has L. 900: Required my stock at first? *Answ.* L. 1300.

Here let my stock be a , and after 10 years, $3a$; subtract my loss, and double the remainder; which divide by 6, you have this equation, $\frac{6a-2400}{6} = 900$.

Multiply by 6, $6a-2400=5400$ Add 2400, $6a=7800$ Divide by 6, $a=1300$, his stock.	Or thus: $\frac{6a-2t}{6} = n$ $6a-2t=6n$ $6a=6n+2t$ $a=\frac{6n+2t}{6}$
---	---

Here t represents 1200, and n 900, and $6n+2t$ at last must be brought to numbers, and divided by 6. In both these operations all the steps of Reduction might be done at once.

Prob. 3. A man leaves his eldest son half his estate, wanting L. 180, his second son one-third and L. 60 more, and the youngest got half the share of the eldest: Required the estate? *Answer,* L. 2520.

Let

Let the estate be a , then the share of the eldest son $\frac{1}{2}a - 180$, of the next $\frac{1}{3}a + 60$, of the youngest $\frac{1}{4}a - 90$; all these added, make $\frac{1}{12}a - 210$, which is the whole estate, viz. a ; therefore $\frac{1}{12}a - 210 = a$; multiply both sides by 12 to take away the fraction, $13a - 2520 = 12a$; subtract $12a$ from both sides, and you have $a - 2520 = 0$; remove 2520 to the other side by addition, and you have $a = 2520$, the estate.

Prob. 4. What two numbers are those, whose product is triple their sum, and whose sum is triple their difference? Answer, 9 and $4\frac{1}{2}$.

Here there are two quantities required, and two equations given, viz. Their product is triple their sum, whence this equation $ae = 3a + 3e$, and their sum is triple their difference, whence another $3a - 3e = a + e$; these two equations will stand, and be reduced as under:

$$ae = 3a + 3e$$

$$ae - 3a = 3e$$

$$a = \frac{3e}{e-3}$$

$$3a - 3e = a + e$$

$$2a = 4e$$

$$a = 2e$$

Having found two values of a , that is, $a = \frac{3e}{e-3}$, and $a = 2e$; set these values in another equation, thus:

$$2e = \frac{3e}{e-3}$$

Multiply by $e-3$ $2e^2 - 6e = 3e$

Transpose $2e^2 = 9e$

Divide by $2e$ $e = 4\frac{1}{2}$

And having found e , a is known from any of the former equations, it being equal to $2e$.

Prob. 5.

Prob. 5. Bartered 40 yards broad cloth, for 50 yards velvet, and got in balance, the difference of the prices, multiplied by the difference of the yards; bartered again 50 yards of the same cloth, for 60 yards velvet, and then received as balance 10 times the product of the two prices: What was the price of a yard of each?

Answer, Cloth 13s. 4d. velvet 10s.

Call the prices a and e , and from the two bargains we have these two equations.

$$\begin{array}{rcl}
 40a = 50e + 10a - 10e & & 50a = 60e + 10ae \\
 \text{Or } 30a = 40e & & 50a - 10ae = 60e \\
 \text{Or } a = \frac{4e}{3} & \text{Divide by 10} & 5a - ae = 6e \\
 & & a = \frac{6e}{5-e}
 \end{array}$$

Having thus gotten two values of a , set them together, and we have an equation that involves but one unknown quantity, which therefore will also be known.

$$\begin{array}{rcl}
 & 4e & 6e \\
 & \underline{\quad} & \underline{\quad} \\
 & 3 & 5-e \\
 \text{Multiply by the divisors} & 20e - 4e^2 = 18e & \\
 \text{Transpose} & 4e^2 + 18e = 20e & \\
 \text{Subtract } 18e, & 4e^2 = 2e & \\
 \text{Divide by } e, & 4e = 2 & \\
 \text{Divide by 4,} & e = .5 \text{ or } 10s. &
 \end{array}$$

And e being found, a may be known from the former equations. The work might have been much shortened, but for plainness.

Prob. 6. There are 4 arithmetical progressionals, their sum is 36, and the sum of their squares 404: What are the numbers? Answer, 3, 7, 11 and 15.

Let

Let a = the first term, and e the common excess, then the terms will be $a, a+e, a+2e, a+3e$, and these added, make $4a+6e=36$; square both sides of this, you will find it $16a^2+48ae+36e^2=1296$. Now the sum of their squares was given; therefore square all the terms, $a^2, a^2+2ae+e^2, a^2+4ae+4e^2, a^2+6ae+9e^2$; these added, $4a^2+12ae+14e^2=404$; subtract the first equation squared, from four times this last, thus:

$$16a^2 + 48ae + 56e^2 = 1616$$

$$16a^2 + 48ae + 36e^2 = 1296$$

Divide by 20, $20e^2 = 320$
 $e^2 = 16$

$$e^2 = 16$$

So the square root of 16, which is 4, is the common excess; and take 6 times the excess from their sum, $\frac{1}{4}$ of the remainder is the first term, as you may gather from the first equation, and from a consideration of the whole, you may deduce this rule. Subtract the square of the sum of the numbers from the quadruple of the sum of their squares, divide the remainder by 20, quotes the square of the common excess.

Prob. 7. Of 4 arithmetical progressions, there is given the product of the mean terms 165, and the product of the extremes 133 : Required the numbers?

Answer, 7, 11, 15, 19.

Call the first term a , and e the excess, so will the terms be $a, a+e, a+2e, a+3e$.

So the product of the means $a^2 + 3ae + 2e^2 = 165$

The product of the extremes $a^2 + 3ae = 133$

Subtract $2e^3 = 32$

And $e^2 = 16$

The

The excess is found to be 4, but to resolve the other equation $a^2 + 3ae = 113$, you must stay till you learn Quadratic Equations.

Prob. 8. My wives, children, and grandchildren are geometrical proportionals; the number of wives I have had, is as far under 12, as that of my children are above it, and the number of my children are under 60, as much as my grandchildren are above it: Required the number of each?

Ans. 4 wives, 20 children, 100 grandchildren.

Three numbers are required here, and three equations given: The first is from their being three geometrical proportionals, in which the product of the extremes is equal to the square of the mean; therefore, calling the wives a , the children e , and the grandchildren y , we have this equation $ay = e^2$. Again, his wives are under 12 as much as his children are above it, therefore $12 - a = e - 12$; and his children being under 60 as much as his grandchildren were above it, gives this third equation $60 - e = y - 60$. These may stand as under:

$$\begin{array}{lll} ay = e^2 & 12 - a = e - 12 & 60 - e = y - 60 \\ a = \frac{e^2}{y} & a + e = 24 & y + e = 120 \\ & a = 24 - e & y = 120 - e \end{array}$$

Now take the two values of a that are found, and set them in an equation by themselves, thus:

$$\begin{array}{ll} \frac{e^2}{y} = 24 - e & 120 - e = \frac{e^2}{24 - e} \\ e^2 = 24y - ye & 2880 - 144e + e^2 = e^2 \\ y = \frac{e^2}{24 - e} & 144e = 2880 \\ & e = 20 \end{array}$$

Having

Having found another value of y , we set it with that formerly found, so we had at last an equation, which had no unknown quantity in it but e , and therefore it being reduced, e comes to be known; so that whatever equations are given in a question, we must always find one that involves but one unknown quantity, and here e being known, y is easily known, from what went before, where we had $y = 120 - e$; therefore take e , or 20, from 120, remains 100 for y , &c.

Prob. 9. A man, who met a company of beggars, finds, that if he gives 1d. to every lame, and 2d. to every blind man, he will have 11d. over; but if he gives 2d. to every lame, and 3d. to every blind man, he will want 1d. and if he gives 3d. to each of them all, he will want 6d. How many beggars did he meet, and how many pence had he?

Answer, 5 lame, 7 blind; he had 30d.

Let a be the pence, e the blind, and y the lame men, and the work will stand thus:

$$\begin{array}{lll} y + 2e = a - 11 & 2y + 3e = a + 1 & 3y + 3e = a + 6 \\ a = y + 2e + 11 & a = 2y + 3e - 1 & a = 3y + 3e - 6 \end{array}$$

From these three values of a raise two equations; one of which, when reduced, subtract from the other, and you have the value of y .

$$\begin{array}{lll} 3y + 3e - 6 = y + 2e + 11, & 2y + 3e - 1 = y + 2e + 11 & \text{from } 2y + e = 17 \\ 2y + e = 17 & y + e = 12 & \text{take } \underline{y + e = 12} \\ & & y = 5 \end{array}$$

Prob. 10. To find three numbers in geometrical proportion, whose sum may be 70, and the sum of their squares 2100? Answer, 10, 20, 40.

Let the numbers be a, e, y , let their sum be s ; and the sum of their squares S , the work will stand thus:

H h

$$\begin{array}{rcl}
 a+y+e=s & ay=e^2 & a^2+y^2+e^2=S \\
 a+y=s-e & ay=e^2 & a^2+y^2=S-e^2 \\
 a+y=s-e & 2ay=2e^2 & \\
 \hline
 a^2+2ay+y^2=s^2-2se+e^2 & & \\
 2ay & = & 2e^2 \text{ Subt.} \\
 \hline
 a^2 & +y^2=s^2-2se-e^2 & \\
 a^2 & +y^2=S & -e^2 \text{ Subt.} \\
 \hline
 0 & =s^2-S-2se & \\
 & =\frac{s^2-S}{2s} & \\
 c & = &
 \end{array}$$

The first equation was the sum of the quantities equal to s , which, after transposing for convenience, we squared, and from this we subtracted the double of the second equation, and again the third; so we have gotten $e = \frac{s^2 - S}{2s}$, which we must bring to numbers, so will the value of e be found. The second equation was found, by calling, the product of the extremes equal to the square of the mean or middle number, which holds true in all geometrical proportionals. A rule or canon deduced from the above work would be this, From the square of the sum of the numbers given subtract the sum of their squares, divide the remainder by double their sum, quotes the middle number, then you have the sum of two numbers, and sum of their squares. To find them out see Prob. 1. next Section.

Prob. 11. Three persons, who had about them considerable sums of money, meet with three bands of robbers; the first takes from A and B half their money, and from C 100 guineas, leaving them as much as A had at first; the second takes from A and C half what they had left, and from B 100 guineas, then they had as much left as B had at first; the third takes from B and C half what they had, and from A 100 guineas,

guineas, and now they had as much left as C had at first: Required what each had at first?

Answer, A, L. 1200, B, L. 660, and C, L. 375.

Be careful to raise the equations. It would require too many words to illustrate them here. They will stand as under.

$$\frac{1}{2}a + \frac{1}{2}e + y - 105 = a$$

$$\frac{1}{2}a + \frac{1}{2}e + \frac{1}{2}y - 157.5 = e$$

$$a + e + 2y - 210 = 2a$$

$$a + 2e + 2y - 630 = 4e$$

$$a = e + 2y - 210$$

$$a = 2e - 2y + 630$$

$$\frac{1}{2}a + \frac{1}{2}e + \frac{1}{2}y - 183.75 = y$$

$$a + e + y - 735 = 4y$$

$$a = 3y - e + 735$$

These may be called *equations of the first order*, and from them raise the two following, of the *second order*.

$$2e - 2y + 630 = e + 2y - 210 \quad e + 2y - 210 = 3y - e + 735$$

$$e = 4y - 840$$

$$2e = y + 945$$

$$e = \frac{y + 945}{2}$$

And from these two raise an *equation of the third order*, which will involve but one unknown quantity, and so you come to a resolution.

$$4y - 840 = \frac{y + 945}{2}$$

$$8y - 1680 = y + 945$$

$$7y = 2625$$

$$y = 375$$

Prob. 12. The horses, hogs, and cows of a certain farmer are geometrical proportionals, and their sum is the square of the horses; again, his hogs, cows and sheep are arithmetical proportionals, and their product, divided by their sum, quotes the square of his hogs: What number of each?

Ans. 7 horses, 14 hogs, 28 cows, and 42 sheep.

Let

Let a be the first number, and e the ratio of the progression, then will a be the horses, ae the hogs, and ae^2 the cows, and the sum of these $a+ae+ae^2=a^3$, and this reduced, is $a=e^2+e+1$. Again, as the sheep must exceed the cows as much as they exceed the hogs, the excess must be the second term, multiplied by the ratio wanting 1: so the sheep are $2ae^2-ae$; which multiplied by the hogs and cows, the product is $2a^3e^3-a^3e^2$, and this divided by their sum, quotes the square of the hogs, a^2e^2 : so $\frac{2a^3e^3-a^3e^2}{3ae^2}=a^2e^2$; and this multiplied by $3ae^2$, gives $2a^3e^3-a^3e^2=3a^3e^4$, and $2e^3-e^2=3e^4$, and $e=2$ the ratio; the rest is easy.

Prob. 13. The lords, knights, and farmers of a certain district, 120 in number, are to pay a public contribution; the lords pay the half of it, at L.5 each, the farmers $\frac{1}{5}$ of it, at 6s. 8d. each, and the knights the rest of it, at L.2 each: I demand what the sum is, with the number of every rank?

Answer, The sum L.200, the lords L.20, the knights 40, the farmers 60.

Let u represent the money, and a, y, e the lords, &c. then putting $120=n$, we have $a+y+e=n$, or $a=n-y-e$, and from the parts they paid, we have $5a=\frac{1}{5}u$, or $a=\frac{u}{10}$, and $2y=\frac{2}{5}u$, or $y=\frac{u}{5}$, and $\frac{1}{3}e=\frac{1}{10}u$, or $e=\frac{3u}{10}$; from the first and second of these equations we have $n-y-e=\frac{u}{10}$, or $e=\frac{10n-10y-u}{10}$; and from this and the third we have $\frac{3u}{10}=\frac{10n-10y-u}{10}$, or $y=\frac{10n-4u}{10}$; and from this and another foregoing

we

we have $\frac{w}{5} = \frac{10n-4n}{10}$, or when reduced, $n = \frac{5n}{3}$; so 5 times n being 600, a third of it is 200 the money.

Prob. 14. A cook went to market and bought 5 stones of beef, and 3 of mutton, and saved of his money 5s. another, with double his money, buys 8 stones of beef, and 9 of mutton, and reserves of his money 8s. a third, with three times the money, buys 12 stones of beef, and 14 of mutton, and brings home of his money 10s. I demand the money each had, with the price of the beef and mutton?

Ans. The first had 42s. &c. beef 5s. mutton 4s.

Let the money of the first be a , a stone of beef e , and of mutton u .

$$a = 5e + 3u + 5 \quad 2a = 8e + 9u + 8, \text{ or } a = \frac{8e + 9u + 8}{2}$$

$$3a = 12e + 14u + 10, \text{ or } a = \frac{12e + 14u + 10}{3} \quad \text{These are equations of the first order.}$$

$$5e + 3u + 5 = \frac{8e + 9u + 8}{2} \text{ this reduced } e = \frac{3u - 2}{2}$$

$$\text{Again, } 5e + 3u + 5 = \frac{12e + 14u + 10}{3} \text{ this reduced, is}$$

$$e = \frac{5u - 5}{3} \quad \text{And these two last put together } \frac{5u - 5}{3}$$

$$= \frac{3u - 2}{2}, \text{ and reduced, gives } u = 4.$$

Prob. 15. A, B and C have among them 100 guineas; A changes his into half-guineas, B his into shillings, and they have among them all 955 pieces; again, A changes his into quarter-guineas, and C his into sixpences,

pences, and then they have in all 1270 pieces: How many guineas had each?

Answer, A had 55, B 40, and C 5 guineas.

Let the numbers of guineas be a, e, y , let $c=100$, $n=955$, and $T=1270$, $a=c-e-y$.

$$\begin{array}{rcl} 2a+21e+y=n & & 4a+21e+42y=T \\ n-21e-y & & T-21e-42y \\ \text{and } a=\frac{\quad}{2} & & \text{and } a=\frac{\quad}{4} \end{array}$$

We need not now set down every step of Reduction, the learner by this time being supposed to be able to do something of himself. Now, from the three equations above find other two.

$$\begin{array}{rcl} c-e-y & = & \frac{n-21e-y}{2} \\ c-e-y & = & \frac{T-21e-42y}{4} \\ c & = & \frac{n-2e+y}{19} \\ c & = & \frac{T-4e-38y}{17} \end{array}$$

And setting these two last equations together, $\frac{n-2e+y}{19} = \frac{T-4e-38y}{17}$, and reduced, you have $y = \frac{19T-42e-17n}{739}$, and bringing these to numbers, by multiplying, subtracting and dividing, you find $y=5$.

Prob. 16. Four men have each a certain sum of money: the first says to the other three, Give me $\frac{1}{2}$ your money, and L. 1 more, and I shall have L. 1000; the second says to the other three, Give me $\frac{1}{3}$ your money, and L. 1 more, I shall have L. 1000; the third says to the rest, Give me $\frac{1}{4}$ of your money, and L. 1 more, I shall have L. 1000; and the fourth in like manner says, Give me $\frac{1}{5}$ of your money, and L. 1 more, I shall have L. 1000:

L.1000: Now I demand what sum of money had each?

Anfw. The first L.27, the second L.513, the third L.675, and the fourth L.756.

Let their sums be a, e, u, y ; let n be 999, and you will find these equations of the first order when reduced.

$$a = \frac{2n - e - u - y}{2} \quad a = 3n - 3e - u - y \quad a = 4n - 4u - e - y$$

$$a = 5n - 5y - e - u$$

Out of these you might have six equations, if you would, but you have use only for three, which may be different ways had, only you must make use of all those of the former rank, or order, in finding those of a second: Here comparing the first of the above with all the three that follow it, you have these three that follow when reduced.

$$e = \frac{4n - u - y}{5} \quad e = 6n - 7u - y \quad e = 8n - 9y - u$$

And from these in like manner the two following.

$$u = \frac{13n - 2y}{17} \quad u = 9n - 11y \quad \text{and} \quad \frac{13n - 2y}{17} = 9n - 11y$$

$$\text{and } y = \frac{140n}{185}, \text{ or } y = 756$$

Prob. 17. Four persons thus discourse of their money; says the first to the others, Give me half your money, I shall have L.700; says the second, Give me half your money, I shall have L.800; says the third, Give me half your money, I shall have L.900; and says the last to the rest, Give me half your money, I shall

shall have L. 1000: Now, I demand the money of each?

Ans. The first L. 40, the second L. 240, the third L. 440, and the fourth L. 640.

Prob. 18. A dealer buys 400 stones of butter and cheese, for L. 100, paying 5s. a-stone more for his butter than his cheese, and afterwards sells $\frac{1}{4}$ of his butter, and $\frac{3}{4}$ of his cheese, for L. 62 14, gaining 10 per cent. : I demand how many stones of each he bought, with the prices?

Answer, 80 of butter, at 9s. 320 of cheese, at 4s.

Let a be the stones of butter, e the stones of cheese, y the price of a stone of butter, $y-5$ a stone of cheese, $4e=400$, $20c=L. 100$, considering it as shillings, $s=1140s.$ or L. 57, for L. 62 14, at 10 per cent. gain, will be L. 57 first cost.

$$a+e=4c$$

$$a=4c-e$$

$$ay+ey-5e=20c$$

$$20c+5e-ey$$

$$a=\frac{20c+5e-ey}{y}$$

$$\frac{1}{4}ay+\frac{3}{4}ey-\frac{1}{4}e=s$$

$$ay+3ey-15e=4s$$

$$a=\frac{4s-3ey+15e}{y}$$

$$4c-e=\frac{20c+5e-ey}{y}$$

$$4cy-ye=20c+5e-ey$$

$$y=\frac{20c+5e}{4c}$$

$$4c-e=\frac{4s-3ey+15e}{y}$$

$$4cy-ye=4s-3ey+15e$$

$$4cy+2ey=4s+15e$$

$$y=\frac{4s+15e}{4c+2e}$$

$$4c+2e$$

$$\begin{array}{r}
 41+15e \quad 20e+5e \\
 \hline
 4c+2e \quad 4e \\
 16sc+60ce=80c^2+60ce+10e^2 \\
 10e^2+80c^2=16sc \\
 16sc-80c^2 \\
 e^2=\frac{\quad}{10} \\
 e=320
 \end{array}$$

Prob. 19. Bought 100 gold rings, silver clasps, and brass buckles, weighing in all 70 ounces, for L. 33: I demand how many there are of every kind, a ring weighing 6dwt. value 25s. a clasp 10dwt. value 2s. 6d. a buckle 25dwt. value 15d.?

Here let a represent the number of rings, e the clasps, and y the buckles; also $c=100$, p = the pennyweights in 70 ounces, and $T=33$, $a=c-e-y$, and considering the weights $6a=10e=25y=p$, and from the values, considering them as fractions of a pound, $1\frac{1}{4}a+\frac{1}{8}e+\frac{1}{16}y=T$; and these being reduced, our equations of the second order will be

$$\begin{array}{r}
 p-10e-25y \\
 c-e-y=\frac{\quad}{6} \\
 \text{And } e=\frac{p-19y-6c}{4}
 \end{array}
 \qquad
 \begin{array}{r}
 16T-2e-y \\
 c-e-y=\frac{\quad}{20} \\
 e=\frac{20c-19y-16T}{18}
 \end{array}$$

And these set together and reduced, make $y=\frac{64T+18p-188c}{266}$ and $y=32$. So the rings are 20, the clasps 48, and the buckles 32.

Prob. 20. Bought a dozen of eggs, a score of apples, and an hundred plumbs, for 7d. next day bought 3 dozen eggs, 4 score apples, and 500 plumbs, for 2s. 3d. and again, 10 dozen eggs, 16 score of apples,
I i

ples, and 2000 plumbs, always at the same rate, for 8s. 6d. What was the rate of each?

Answer, A dozen eggs, 3d. a score of apples, 2d. an hundred plumbs 2d.

Prob. 21. Hired a carpenter for 30 days, a shoemaker for 20, and a taylor for 10, their wages came to be 23 half-crowns; again employed the carpenter for 20, the shoemaker for 10, and the taylor for 30 days, their wages 19 half-crowns; lastly, having the carpenter 10, the shoemaker 30, and the taylor 20 days, their wages were 18 half-crowns: I demand the wages of each per day?

Ans. The carpenter 15d. the shoemaker 9d. the taylor 6d.

Prob. 22. Bought a shilling's worth of apples, to distribute among a company of ladies and boys, and proposing to give each lady 10, and each boy 6, I find two ladies and a boy will be unserved; bought another shilling's worth, and then giving each lady 20, and each boy 10, a lady and two boys were unserved; laid out a third shilling, and then made every lady 24, and each boy 13, and all were served: How many apples had I for a shilling? Answer, 90.

Prob. 23. A widow, a wife, and a maid, on comparing their ages, find, That the widow is 20 years older than the wife, that the wife is double the age of the maid, and the product of all their years, divided by their sum, quotes the square of the maid's years: Required their ages?

Answer, The maid 20, the wife 40, and the widow 60.

Prob. 24. Bought 4 horses, A, B, C and D; gave four times the price of D for C, and three times the price of B for A; moreover, the sum of the prices of A and

A and B is equal to the product of C and D, and the product of the first and last, taken from that of the two middlemost, leaves the sum of the two dearest, or product of the two cheapest: I demand the price of each?

Answ. A, L.48, B, L.16, C, L.16, and D, L.4.

Prob. 25. Bought 600 trees, of 8 kinds, for L.350, their numbers were in Arithmetical Progression, viz. Oaks at 30s. ashes at 25s. elms at 20s. planes at 15s. elders at 12s. firs at 8s. beeches at 4s. and birches at 2s. I demand the number of each, the oaks being fewest? Answer, 40 oaks, 50 ashes, 60 elms, &c.

Let a be the first term of the progression, or number of oaks, and e the common excess, so the terms will be $a, a+e, a+2e, \&c.$ to eight terms, all these added, make $8a+28e=600$, and multiply every term by the price, and add the products, you have $116a+236e=7000$.

Prob. 26. A company at a tavern paying 5s. a-piece, fall 5s. short of the reckoning; they then call for bottles a-piece more, and again paying 5s. each, they are 5s. over the reckoning; lastly, they call for bottles a-piece, pay 2s. each more, and all is clear: I demand the number in company, the reckoning, and the price of a bottle?

Answer, 5 in company, the reckoning 30s. at first, a bottle 3s.

Prob. 27. A man would perform a journey in a certain number of hours, and finding, after he had travelled a third part of his time, he had performed but a fourth part of his journey; he travels on 5 hours more, and then finds he has travelled 27 miles in all; after that he doubles his pace, and yet at the end of the time

time is 3 miles short of compleating his journey: What was the journey? Answer, 48 miles.

Prob. 28. At my marriage, my age was triple that of my wife's, whereas it is now only double; we had a child every two years, till we had 10, and now the ages of all our children are equal to our own: What is the age of each?

Answer, The husband's age 110, the wife's 55, the youngest child $7\frac{1}{2}$, and the eldest $25\frac{1}{2}$.

Prob. 29. A number consisting of two digits, whereof the last is double the first, is such, that the sum of its digits, their product, and the number itself, are geometrical proportions: What is the number?

Answer, 36.

Prob. 30. There is a number of two figures, the sum of its digits is the square root of it, and if the figures be inverted, they make a number double the sum of its digits: Required the number?

Answer, 81.

Prob. 31. In a bee-hive there are four working bees for every drone, and their numbers are such, that the square root of the drones, the square root of the working bees, and the hundredth part of the whole, are three arithmetical proportionals: Required the number of each? Answer, 3600 drones, 14400 bees.

Prob. 32. Bought 50 pieces serges, and 60 of kerseys; the latter L. 5 per piece dearer than the former; sold 50 pieces of both for as much as I paid for the whole, and thus far find my gain 12 per cent. I demand the price of a piece?

Answer, Serges L. 10, kerseys L. 15 per piece.

Prob. 33. Three equal fields of grafs are to be cut down by two men, A and B, who working both together, do the first piece in 12 days; at the second plot,
A works

A works 12 days alone, and afterwards $4\frac{2}{3}$ days with B to finish it: At the third plot, B works 12 days alone, and again 7 days with A, and yet there was 72 falls uncut: Required the content of a lot, and what each mowed in a day?

Ans. 27 acres a-lot, A 216, B 144 falls a-day.

Prob. 34. Sold three horses for L. 107: Having gained 20 per cent. on the first, 40 on another, and 60 on the last, and $42\frac{2}{3}$ per cent. on the whole, I demand the price of each horse, the second having cost me L. 5, and the third L. 10 dearer than the first?

Answer, 24, 35, and L. 48.

Prob. 35. A man, who had 10 children born at equal intervals, said, That the sum of the ages of the 5 eldest, divided by that of the 5 youngest, would quote the age of the second child; and the sum of the ages of the 2 eldest, divided by that of the 2 youngest, would quote the age of the fifth child: Required the childrens ages? Answer, 1, 3, 5, &c. to 19 years.

Prob. 36. Bought three pieces of cloth, and paying for each as many shillings per yard, as the number of yards was in both the other two pieces, the first cost me L. 4 5, the second 5 guineas, and the third L. 6: I demand how many yards are in each piece?

Answer, 5, 7, and 9 yards.

Prob. 37. What two numbers are these whose difference is to their sum as 2 to 3, and their sum to their product as 3 to 5? Answer, 10 and 2.

Prob. 38. What two numbers are these whose sum multiplied by their difference produces 81, and the sum of their squares by the difference 29889?

Answer, 12 and 15.

This is the same as if the difference of their squares and of their biquadrates were given.

Prob. 39.

Prob. 39. What fraction is that to whose numerator if you add 1, its value will be $\frac{2}{3}$; but if you increase the denominator by 1, it will be $\frac{1}{4}$?

Answer, $\frac{1}{12}$.

Prob. 40. What number added to 15, 24 and 45, will make them all three geometrical proportionals?

Answer, 9.

Prob. 41. A son's age is the square root of the mother's, and the cube root of the father's, and half the mother's is the square root of the father's: Required the ages?

Answer, 4, 16, 64.

Prob. 42. Three merchants put in L. 700 into company for trade, of which C's part was L. 200; they gained in all L. 140, of which A's share was L. 56: I demand stock and gain of each?

Answer, A's stock L. 280, C's gain L. 40, &c.

Prob. 43. In a company at a banquet, the gentlemen in number are double the ladies, and the cube of the number of ladies is triple the square of the gentlemen: Required the number of each?

Answer, 24 gentlemen, 12 ladies.

Prob. 44. A sum of money being at interest as many years as there were pounds, had amounted to L. 240; and half the sum being at interest for half the time, amounted to L. 75: I demand the rate of interest?

Answer, 5 per cent.

Prob. 45. There are two numbers; the first, with half the second, is equal to their product; and the second, with a third the first is also equal to their product: What are the numbers?

Answer, 1.25, 1.66.

Prob. 46.

Prob. 46. A number consisting of three digits in Arithmetical Progression is such, that if you divide the number by the sum of its digits, the quotient is 26; and if you add 396 to the number, the figures will be inverted: *Queritur*, the number?

Answer, 468.

Prob. 47. Bought 50 pieces serges, and 60 of kerseys, the latter L. 5 per piece dearer than the former; I sold 50 pieces of each for as much as I paid for the whole, and so far as sold, my gain is 12 per cent. but if I sell my other 10 of kerseys as before, my gain will then be but 10 per cent. I demand the prices?

Answer, Serges L. 10, kerseys L. 15; sold both at L. 14: So there is loss on the kerseys.

Prob. 48. Bought a parcel of hats and bonnets; the price of a hat 10s. more than that of a bonnet; got a bonnet to the dozen, and a hat to the score; the bonnets in number were to the hats as 13 to 24, and their amounts as 7 to 80; gained 3s. on every hat, and 6d. on every bonnet, and then the amounts as 13 to 144: Required the number of each, and the prices?

Answer, 168 hats, at 12s. but paid only for 160; 91 bonnets, at 2s. paid but for 84.

II. QUADRATIC EQUATIONS.

Many equations resulting from the foregoing problems, will have in them a square of a quantity; but we use only to call those *quadratic equations*, which, besides the square of the unknown quantity, involve also the simple power of it, and therefore are said to be of two dimensions, as in this equation, $6a^2 - 48a + 72 = 0$, or, which is the same thing, $6a^2 - 48a = -72$, where we have two dimensions of the unknown quantity,

tity, and where the square is called the *first term*, or highest dimension, the simple power the *second term*; and to resolve such equations proceed as under :

Bring both the dimensions to stand on one side of the equation, generally the left side, and remove the known quantities to the contrary side ; if any co-efficient be prefixed to the square of the unknown quantity, divide every member of both sides by it, so that the square may stand without any co-efficient ; next add to both sides of the equation the square of half the co-efficient of the second term, that is, the simple power, or, which is all one, the fourth part of the square of that co-efficient; extract the square root on both sides, which, on the one side, will be the letter that represents the unknown quantity, with half the same co-efficient of the second term annexed to it ; the other side you bring to numbers, and extract the root as usual, and when found, will stand with a twofold sign before it ; for this root may be either positive or negative ; then transpose the foresaid half co-efficient to the known side, where the known square root stands, and their sum, or difference, will be equal to the unknown quantity, for all quadratic equations admit of two answers ; one of which, according to the nature of your question, will sometimes be nonsense, but you take that which answers your purpose.

Now, to resolve the equation given above, $6a^2 - 48a = -72$: Divide all the parts by 6, and you have $a^2 - 8a = -12$; add to both sides the square of 4, $a^2 - 8a + 16 = 4$, for 12 on the known side was negative; now extract the root on both sides, $a - 4 = \pm 2$; transpose 4, and you have $a = 4 \pm 2$, so a is either 6 or 2, either of which numbers put in the equation for it, will be found to answer.

The reason why a square root is either positive or negative may be understood from the rules of Multiplication,

plication, that, like signs, give an affirmative product; for either an affirmative or negative quantity being squared, gives an affirmative power; but if a square quantity be negative, its root is impossible, and therefore if in any problem you have to extract the square root of a negative, such a problem is capable of no rational solution.

The reason of adding the square of half the co-efficient of the second term in a quadratic equation, is to make a complete square; for if you square the root $a-4$ in the above equation, you find it to produce $a^2-8a+16$, the same whose root you extracted. It may also be observed, that this squared half co-efficient may well enough be omitted on the unknown side, except for form's sake, the root being well enough known without it, and we omit it always in the following problems.

Prob. 1. The sum of two numbers is 34, and the sum of their squares 650: Required the two numbers?

Answer, 11, and 23.

Let the two numbers be a and e , then $a+e=34$, and $a=34-e$; square this, and you have $a^2=1156-68e+e^2$: another equation from the sum of their squares is $a^2+e^2=650$, and $a^2=650-e^2$. Now, $1156-68e+e^2=650-e^2$, for both are equal to a^2 ; then by transposition $2e^2-68e=-506$; divide by 2, $e^2-34e=-253$; add the square of 17, $e^2-34e+36$; extract the root, $e-17=6$; transpose 17, $e=17+6$, or $+6$: So is e 11 or 23; but whatever you call e , the other will be a .

Or putting symbols also for known quantities, the work may stand thus: Let the sum of the numbers be s , and the sum of their squares S .

K k

$$a+e=s$$

$$\begin{array}{rcl}
 a+e=s & a^2+e^2=S & s^2-2se+e^2=S-e^2 \\
 a=s-e & a^2=S-e^2 & 2e^2-2se=S-e^2 \\
 a^2=s^2-2se+e^2 & & \frac{S-e^2}{2} \\
 & & e^2-2se=\frac{S-e^2}{2} \\
 & & e^2-se+\frac{1}{4}s^2=\frac{S-e^2}{2}+\frac{1}{4}s^2 \\
 & & e-\frac{1}{2}s=\sqrt{\frac{S-e^2}{2}+\frac{1}{4}s^2}
 \end{array}$$

From this is derived the rule given, Prob. 12. of Extraction of Roots, and all these problems may be proved such ways as this, and the learner may invent numberless examples from them for practice.—See the 14th problem.

Prob. 2. The sum of two numbers is 17, and the sum of their cubes 1547: Required the numbers?

Answer, 6, and 11.

Call the numbers a , and e , then $a+e=17$, or $a=17-e$; also $a^3+e^3=1547$, or $a^3=1547-e^3$; let the first equation be cubed, $a^3=4913-867e+51e^2-e^3$; so is $4913-867e+51e^2-e^3=1547-e^3$; throw away e^3 , and transpose $51e^2-867e=-3366$; divide by 51, $e^2-17e=-66$; add the square of half 17, $e^2-17e+6.25=-66+6.25$; extract the root, $e-8.5=2\frac{1}{2}$, and $e=6$, or 11.

Prob. 3. The product of two numbers is 36, and the sum of their cubes 793: What are the numbers?

Answer, 4, and 9.

$$\begin{array}{rcl}
 \text{Here } ae=36 & a^3+e^3=793 & 46656 \\
 \text{And } a=\frac{36}{e} & a^3=\frac{793-e^3}{e^3} & \frac{46656}{e^3}=793-e^3 \\
 \text{Cubed } a^3=\frac{46656}{e^3} & \text{Multiply } 46656=793e^3-e^6 & \\
 & \text{Transpose } e^6-793e^3=-46656 & \\
 & \text{Add square of half } 793 & e^6-793e^3+110556.25 \\
 & \text{Extract the root } e^3-396.5=332.5 & \\
 & \text{And } e^3=729, \text{ or } 64. &
 \end{array}$$

So

So you have got the cube of your number, instead of the number itself; for this was the same as a quadratic equation, the third and sixth having the same connection as the square and simple power; the same holds also if an equation involves the second and fourth powers, when you will get the square of your answer, instead of the answer itself. We may also observe here, that the powers of quantities are multiplied by adding the exponents of the powers, and divided by subtracting them; thus a^4 , multiplied by a^4 , would give a^8 , &c. The above done more handsomely, thus putting p for the product, and s for the sum of the cubes.

$$\begin{array}{lcl} ae = p & a^3 + e^3 = s & \frac{p^3}{e^3} = s - e^3 \\ a = \frac{p}{e} & a^3 = s - e^3 & p^3 = se^3 - e^6 \\ & & e^6 - se^3 = -p^3 \\ & & e^6 - se^3 = \frac{1}{4}s^2 - p^3 \\ & & e^3 - \frac{1}{2}s = \sqrt{\frac{1}{4}s^2 - p^3} \\ & & e^3 = \frac{1}{2}s \pm \sqrt{\frac{1}{4}s^2 - p^3} \end{array}$$

Prob. 4. Sold a horse for L.39, and gained per cent. as much as the horse cost me: What did the horse cost? Answer, L.30.

Call the price of the horse a ; then state four proportional numbers, thus: $a : 39 :: 100 : 100 + a$; multiply extremes and means, you have $a^2 + 100a = 3900$; add the square of 50, and extract the root on both sides, you have $a + 50 = 80$, or $a = 30$: the negative root is here useless. You might have stated a different way at first, saying, As the price of the horse to the gain, so 100 to the price of the horse, or gain per cent. $a : 39 - a :: 100 : a$; multiply extremes and means, $a^2 = 3900 - 100a$, and $a^2 + 100a = 3900$: The same as before, or putting s for the selling price, and e for 100, work thus:

$$a : s$$

$$\begin{array}{lcl}
 & a : s :: c : c+a \\
 \text{Multiply} & a^2 + ca = sc \\
 \text{Add square } \frac{1}{2}c, & a^2 + ca = sc + \frac{1}{2}c^2 \\
 \text{Extract} & a + \frac{1}{2}c = \sqrt{sc + \frac{1}{2}c^2} \\
 & a = \sqrt{sc + \frac{1}{2}c^2} - \frac{1}{2}c
 \end{array}$$

Bringing $sc + \frac{1}{2}c^2$ to numbers, extract the square root as usual, which you have always to do, at the resolution of an equation.

Prob. 5. What is the hour of the day, when the time passed from midnight is the square of the time since noon? Answer, 4 afternoon.

$$\begin{array}{lcl}
 \text{Here,} & 24 - a^2 = 12 - a \\
 \text{Transpose} & a^2 - a + 12 = 24 \\
 \text{Subtract 12} & a^2 - a = 12 \\
 \text{Add .5 squared} & a^2 - a = 12.25 \\
 \text{Extract} & a - .5 = 3.5 \\
 & a = 4
 \end{array}$$

Prob. 6. Bought two carcafes of beef, weighing 32 stoncs; the first cost 56s. the second L. 3, yet the last a halfpenny a-pound cheaper than the other: What was the weight of each, and price per lb. or per stone?

Answer, 14stones, at 4s. and 18 stoncs, at $2\frac{1}{2}$ d. per lb.

Let the price of a stone be a , and the other $a - \frac{2}{3}$.

$$\begin{array}{lcl}
 & 56 & 60 \\
 & - + - & = 32 \\
 & a & a - \frac{2}{3} \\
 \text{Multiply by both divisors,} & 56a - 37\frac{1}{3} + 60a = 32a^2 - 21\frac{1}{3}a \\
 \text{And by transposition,} & 32a^2 - 137\frac{1}{3}a = -37\frac{1}{3} \\
 \text{Divide by 32,} & a^2 - 4\frac{7}{4}a = -1\frac{1}{8} \\
 \text{Add} & a^2 - 4\frac{7}{4}a = -3.4379, \text{ \&c.} \\
 \text{Extract} & a - 2.145 = 1.854 \\
 & a = 4
 \end{array}$$

Prob. 7. Two ships sail round the world, the one two miles a-day more than the other, and performs the

the voyage 45 days sooner: I demand what miles each failed in a day?

Answer, One 32, the other 30 miles a-day.

Let their miles be, the one a , the other $a+2$, and let m be 21600:

$$\text{Then} \quad - \quad - \quad - \quad \frac{m}{a} = \frac{m}{a+2} + 45$$

$$\text{Multiply by divisors,} \quad - \quad ma + 2m = ma + 45a^2 + 90a$$

$$\text{Throw away } ma, \quad - \quad 2m = 45a^2 + 90a$$

$$\text{Transpose and divide} \quad - \quad a^2 + 2a = \frac{2m}{45}$$

$$\text{Add 1, and extract} \quad - \quad - \quad a+1 = \sqrt{\frac{2m}{45} + 1}$$

$$a = 30$$

Prob. 8. A man makes his will as follows: He leaves to his eldest son John $\frac{1}{2}$ his subject wanting L. 24; to William $\frac{1}{3}$, wanting L. 30; to James $\frac{1}{4}$, wanting L. 70; and to Robert the remainder, which was just the square root of the share of John: I demand the subject? Answer, L. 1200.

Let a be the subject, $c=124$, viz. the sum of 24, 30, and 70; also, $x=24$: then adding all the legacies together, there arises the following equation:

$$\frac{1}{2}a - c + \sqrt{\frac{1}{2}a - x} = a$$

$$\text{Subtract } a, \text{ and transpose} \quad \sqrt{\frac{1}{2}a - x} = c - \frac{1}{2}a$$

$$\text{Square both sides,} \quad - \quad \frac{1}{4}a - x = c^2 - \frac{1}{2}ca + \frac{1}{4}a^2$$

$$\text{Double both sides,} \quad - \quad a - 2x = 2c^2 - ca + \frac{1}{2}a^2$$

$$\text{Multiply by 72,} \quad - \quad 72a - 144x = 144c^2 - 72ca + 36a^2$$

$$\text{Transpose,} \quad - \quad a^2 - 24ca - 72a = -144c^2 + 144x$$

$$\text{Add the square of half the co-efficients of } a, \quad \left. \begin{array}{l} \\ \end{array} \right\} a^2 - 24ca - 72a = 864c^2 + 1296 - 144x$$

$$\text{Extract} \quad - \quad a - 12c - 36 = \sqrt{864c^2 + 1296 - 144x}$$

$$\text{Transpose} \quad - \quad - \quad a = 12c + 36 + \sqrt{864c^2 + 1296 - 144x}$$

So

So a will be found either 1848 or 1200, according as you make the root positive or negative: but the last is found to answer the purpose.

Prob. 9. What two numbers are these whose product is double their sum, and the sum of their squares double their product? Answer, 4 and 4.

It is sometimes commodious to represent a quantity by the product of two quantities multiplied together; so here let one number be a , the other ae , and we have these equations.

$$\begin{aligned} a^2e &= 2ae + 2a \\ ae &= 2e + 2 \\ a &= \frac{2e+2}{e} \end{aligned}$$

$$\begin{aligned} a^2e^2 + a^2 &= 2a^2e \\ e^2 + 1 &= 2e \\ e^2 - 2e &= -1 \\ e - 1 &= 0 \end{aligned}$$

So e is found equal to 1. $2e+2$ divided by 1, is 4 for a ; and multiply this by 1, gives ae , also 4.

Prob. 10. What two numbers are these whose product is double their sum, and the difference of their squares double their product?

Answer, 6.828, and 2.828 nearly.

Just as before.

$$\begin{aligned} a^2e &= 2ae + 2a \\ ae &= 2e + 2 \\ a &= \frac{2e+2}{e} \end{aligned}$$

$$\begin{aligned} a^2e^2 - a^2 &= 2a^2e \\ e^2 - 1 &= 2e \\ e^2 - 2e &= 1 \\ e - 1 &= \sqrt{2} \\ e &= 1.414 \end{aligned}$$

Prob. 11. There are 4 numbers in geometrical progression, the sum of the extremes is 336, and the sum of the means 144: Required the numbers?

Answer, 12, 36, 108, 324.

Notwithstanding 4 numbers are required here, we need but one unknown quantity; for in geometrical proportionals,

proportionals, the square of any term divided by that next it on one side, gives the term next it on the other side. Let then the sum of the means be m , and of the extremes E ; make the second term a , the third will be $m-a$, the first $\frac{a^2}{m-a}$, and the fourth $\frac{m^2-2ma+a^2}{a}$ then the sum of the extremes being equal to E , gives us this equation.

$$\frac{a^2}{m-a} + \frac{m^2-2ma+a^2}{a} = E$$

$$a^3 + m^3 - 3m^2a + 3ma^2 = mEa - Ea^2$$

$$3ma^2 + Ea^2 - 3m^2a - mEa = -m^3$$

Bring these to numbers, it will be plainest.

$$768a^2 - 110592a = -2985984$$

$$a^2 - 144a = -3888$$

$$a - 72 = \sqrt{\frac{5184}{1296}}$$

$$a = 36$$

Whence this curious rule, To triple the sum of the mean, add the sum of the extremes for a divisor, by this divide the cube of the means, subtract the quotient from the square of half the sum of the means, and the square root of this remainder taken from half the means, leaves the second term.

Prob. 12. Bought 120 oxen and cows, for L. 720; paid for every ox as many crowns as the number of oxen, and as many shillings for every cow as the number of cows: Required the number of each?

Answer, 40 oxen, 80 cows.

Let m be the money in shillings, $n=120$, a the oxen, and e the cows. Work as under.

$$a+e=n$$

$$\begin{aligned}
 5a^2 + e^2 &= m & n^2 - 2ne + e^2 &= \frac{m - e^2}{5} \\
 a + e &= n & a^2 &= \frac{m - e^2}{5} \\
 a &= n - e & 5n^2 - 10ne + 5e^2 &= m - e^2 \\
 a^2 &= n^2 - 2ne + e^2 & 6e^2 - 10ne &= m - 5n^2 \\
 & & e^2 - \frac{5}{3}ne &= \frac{m - 5n^2}{6} \\
 e - \frac{5}{6}n &= \sqrt{\frac{m - 5n^2}{6} + \frac{25n^2}{36}} \\
 e &= 80
 \end{aligned}$$

Prob. 13. A merchant sold 180 oxen and 112 cows, for L.2000, giving 5 more cows for L.100 than of oxen for L.120: I demand the prices?

Answer, L.5, and L.8.

$$\begin{aligned}
 112a + 180e &= 2000 & 100 & 120 & 2000 - 180e & 20e \\
 2000 - 180e & & \frac{100}{a} &= \frac{120}{e} + 5 & \frac{2000 - 180e}{112} &= \frac{20e}{e + 24} \\
 a &= \frac{2000 - 180e}{112} & a &= \frac{e}{\frac{120}{e} + 5} & & \\
 & & a &= \frac{e}{\frac{120 + 5e}{e}} & & \\
 & & a &= \frac{e^2}{e + 24}
 \end{aligned}$$

Prob. 14. Three persons, A, B and C, talk thus of their ages: Says A to B, Your age and mine multiplied together, would make up the age of Methuselah; says C to A, Half your years, multiplied by mine, would make up the age of Methuselah; and B to C, One-third your years multiplied by mine, would make up the age of Methuselah: Required the age of each?

Ans. A 25.445+, B 38.124, C 76.249 nearly.

$$\begin{aligned}
 ae &= m & ay &= 2m & ey &= 3m & \frac{2m}{m} &= \frac{m}{\frac{3m}{2}} & \frac{3m}{\frac{3m}{2}} &= 2 \\
 a &= \frac{m}{e} & a &= \frac{2m}{y} & e &= \frac{3m}{y} & e &= \frac{2m}{\frac{3m}{2y}} & y &= 6m \\
 & & & & & & e &= \frac{2}{3}y & &
 \end{aligned}$$

This turns out to be but a simple equation.

Prob. 15.

Prob. 15. The product, sum, and difference of two numbers make 20, and the product, sum, and difference of their squares make 176: Required the numbers? Answer, 4 and 3.

$$ae + 2a = 20$$

$$a^2e^2 + 4a^2e + 4a^2 = 400$$

$$a^2 = \frac{400}{e^2 + 4e + 4}$$

$$a^2e^2 + 2a^2 = 176$$

$$a^2 = \frac{176}{e^2 + 2}$$

$$\frac{400}{e^2 + 4e + 4} = \frac{176}{e^2 + 2}$$

$$400e^2 + 800 = 176e^2 + 704e + 704$$

$$224e^2 - 704e = -96 \text{ \&c. } e = 3$$

Prob. 16. What two numbers are these whose sum, product, and difference of their squares, are all three the same number? Answ. 2.618, and 1.618 nearly.

Let one number be a , the other ae .

$$a^2e = ae + a$$

$$ae = e + 1$$

$$a = \frac{e + 1}{e}$$

$$a^2e = a^2e^2 - a^2$$

$$e = e^2 - 1$$

$$e^2 - e = 1.25$$

$$e - .5 = \sqrt{1.25} \text{ and } e = 1.618$$

Put this value of e in the first equation instead of e ,
 $a = \frac{2.618}{1.618}$, and a will be 1.618, &c.

Prob. 17. There are three numbers; the difference of the first and second is equal to the third; the sum of the first and second is the square of the third; and their product is its cube: Required these numbers?

Answer, 11.089, 6.853, and 4.236 nearly.

$$a - e = y$$

$$a = y + e$$

$$a + e = y^2$$

$$a = y^2 - e$$

$$ae = y^3$$

$$a = \frac{y^3}{e}$$

$$y + e = y^2 - e$$

$$e = \frac{y^2 - y}{2}$$

L 1

$y + e$

$$y + e = \frac{y^3}{e} \quad y^3 - e = \frac{y^3}{e}$$

$$ye + e^2 = y^3 \quad y^2 e - e^2 = y^3$$

Add these two last equations together, they make $y^2 e + ye = 2y^3$, and divide by y , it is $ye + e = 2y^2$, and $e = \frac{2y^2}{y+1}$. Now, we had another value of e above, which set equal to this, and being resolved, the value of y is found 4.236.

$$\frac{y^2 - y}{2} = \frac{2y^2}{y+1}$$

$$y^3 - y = 4y^2$$

$$y^2 - 1 = 4y$$

$$y^2 - 4y = 1$$

Prob. 18. Two merchants join in company for trade; the sum of their stocks is L. 1000; the money of A being in company 5 months, and that of B 7 months, A's proceeds came to be L. 715, and B's L. 639: I demand the stock and gain of each?

Answer, A's stock L. 550, gain L. 165; B's stock L. 450, gain L. 189.

Let A's stock be a , let L. 1000 be m , then B's stock is $m - a$; multiply by their times, and add the products for your first term; let the gain, or g , be the second; A's product the third, his gain, viz. his proceeds wanting the stock, the fourth, as under:

$$7m - 2a : g :: 5a : A - a$$

$$7Am - 2Aa - 7ma + 2a^2 = 5ga$$

Or, $2a^2 - 2Aa - 7ma - 5ga = -7Am$

Bring all to numbers just now, will make least confusion.

$$\begin{array}{r} 2a^2 - 10200a = -5005000 \\ a^2 - 5100a = -2502500 \\ \text{Add } \quad \quad \quad 6502500 \\ \hline a - 2550 = \sqrt{4000000} \\ a = 5550 \end{array}$$

Prob. 19. What two numbers are these whose sum is the square, and whose product is the cube of their difference? Answer, 11.089, 6.236.

This problem, in effect, is the same with the 17th.

Prob. 20. What two numbers are these whose difference added to the difference of their squares, makes 510, and their sum added to the sum of their squares, makes 1350? **Answer,** 20 and 30.

$$\begin{array}{rcl} a - e + a^3 - e^2 & = & 510 \\ a + e + a^3 + e^2 & = & 1350 \\ \hline 2a & + & 2a^3 = 1860 \\ 2a^2 + 2a & = & 1860 \\ a^2 + a & = & 930.25 \\ a + .5 & = & \sqrt{930.25} \\ a & = & 30 \text{ \&c.} \end{array}$$

Prob. 21. The difference of two numbers, multiplied by the difference of their squares, produces 672, and their sum multiplied by the sum of their squares, produces 37380: Required the two numbers?

Answer, 19 and 23.

It is well known the sum of two numbers, multiplied by their difference, gives the difference of their squares, and therefore the square of their difference, multiplied by their sum, will give the product of their difference, multiplied by the difference of their squares: Now, the square of the difference is $a^2 - 2ae + e^2$, and this multiplied by $a + e$, will be 672; and therefore $a^2 - 2ae + e^2 = \frac{672}{a + e}$; also, as the sum of their squares,

multiplied

multiplied by their sum, is 37380, therefore $a^2 + e^2 = \frac{37380}{a+e}$: From the double of this subtract the former.

$$\begin{array}{r} 2a^2 + 2e^2 = 74760 \\ a^2 - 2ae + e^2 = 672 \\ \hline a^2 + 2ae + e^2 = \frac{74088}{a+e} \end{array}$$

Now, $a^2 + 2ae + e^2$ is evidently the square of $a+e$, and if multiplied by $a+e$, the result must be the cube of the same, which will be equal to 74088; therefore extract the cube root of 74088, and you have the sum of the two numbers, viz. 42, by which dividing 37380, you have the sum of their squares, viz. 890; and this is to be resolved by the first problem of this section.

Prob. 22. There are two numbers, their sum is equal to their product, and their sum, or product, added to the sum of their squares, makes 197.9964: Required the numbers? Answer, 13.5 and 1.08.

For their sum put a , then will the sum of their squares be $197.9964 - a$; and if you add double their product to the sum of their squares, you have the square of their sum. Whence this equation:

$$\begin{array}{r} a^2 - a = 197.9964 \\ \quad .25 \\ a - .5 = \sqrt{198.2464} \\ a = 14.58 \text{ their sum.} \end{array}$$

From this find the sum of their squares, and work by problem 1st.

Prob. 23. What two numbers are these whose sum added to their product, makes 119, and the same sum

sum subtracted from the sum of their squares, leaves 182? Answer, 9 and 11.

Just as before, put $a =$ to their sum, then their product is $119 - a$, and the sum of their squares $182 + a$; and to the sum of their squares add double their product, you have the square of their sum; and having found their sum, work by prob. 1st.

Prob. 24. There are three numbers; the product of the first is equal to twice the third, that of the first and third equal to thrice the second, and that of the second and third to quadruple the first: What are the numbers? Answer, 2.4494, 2.8284, 3.4641.

$$\begin{array}{r} ae = 2y \\ 2y \\ a = \frac{2y}{e} \end{array}$$

$$\begin{array}{r} ay = 3e \\ 3e \\ a = \frac{3e}{y} \end{array}$$

$$\begin{array}{r} ey = 4a \\ ey \\ a = \frac{ey}{4} \end{array}$$

$$\begin{array}{r} 2y \quad 3e \\ \frac{2y}{e} = \frac{3e}{y} \\ 2y^2 = 3e^2 \end{array}$$

$$\begin{array}{r} 3e \quad ey \\ \frac{3e}{y} = \frac{ey}{4} \\ 12e = ey^2 \\ y^2 = 12 \end{array}$$

Prob. 25. A, B, C and D have to the value of L.45 about them; A and B have crowns, C and D have guineas; the number of all their pieces are 116; the sum of A and B's crowns, is equal to the product of C and D's guineas; also the gold is to the silver as 7 to 8 with respect to value: How many pieces had each, A having most crowns, and C most guineas?

Answer, A had 60 crowns, B 36, C 12 guineas, D 8.

This might be wrought, no doubt, by a long and tedious train of equations, to be gathered from the *data* in the question, but much easiest by first proportioning the value; for if they had L.45 in all, and
C and

C and D's value, viz. the gold, was to A and B's as 7 to 8, then A and B had L. 24, and C and D, L. 21, and of consequence the crowns were 96, and the guineas 20; and as the product of the guineas is the sum of the crowns, we have only this very simple problem to resolve, viz. the sum of two numbers is 20, and their product 96.

$$\begin{aligned} a + e &= 20 \\ a &= 20 - e \end{aligned}$$

$$\begin{aligned} ae &= 96 \\ 96 & \\ a &= \frac{96}{e} \end{aligned}$$

$$\begin{aligned} 96 & \\ - &= 20 - e \\ e & \\ 96 &= 20e - e^2 \\ e^2 - 20e &= -96 \\ 100 & \\ e - 10 &= \sqrt{4} \\ e &= 8 \text{ or } 12 \end{aligned}$$

The sum of A and B's crowns is 96, but how many each had, matters not.

Prob. 26. There is a heap of apples, pears and plumbs; apples 20dwt. each, at a penny the score, pears 12dwt. each, at a penny the dozen, plumbs 4dwt. each, at 60 a-penny: Now, the apples are half the weight, the pears half the value, and the plumbs half the number of the whole heap: I demand the number of each?

Answer, The apples and pears equal in number, and the plumbs equal them both; for the question admits of many answers.

$$\begin{aligned} 20a &= 12e + 4y \\ 12e + 4y & \\ a &= \frac{12e + 4y}{20} \end{aligned}$$

$$\begin{aligned} \frac{1}{10}a + \frac{1}{30}y &= 12e \\ 5 - y & \\ a &= \frac{5 - y}{3} \end{aligned}$$

$$a = y - e$$

$$\frac{12e + 4y}{20} = \frac{5e - y}{3}$$

$$\frac{5e - y}{3} = y - e$$

$$\begin{aligned} 36e + 12y &= 60 - 20y \\ y &= 2e \end{aligned}$$

$$\begin{aligned} 5e - 3y &= y - e \\ y &= 2e \end{aligned}$$

So all the result you come to is $y = 2e$; therefore call $e = 1$, $y = 2$, a will also be 1. The next two problems of the same kind.

Prob. 27. I have blue cloth, at 11s. six quarters wide; scarlet velvet, at 19s. four quarters wide; and shalloon, at 1s. two quarters wide: Now, the blue is half the quantity of yards, superficial measure, the shalloon is half the number of yards, running measure, and the scarlet half the value: I demand the yards of each?

Answer, 2 yards scarlet, 3 yards blue, 5 yards shalloon.

Or any numbers in the same proportion.

Prob. 28. There is a herd of bulls, cows and hogs; the bulls weigh 39 stones each, and are valued at L. 3; the cows weigh 15 stones, value L. 6; and the hogs weigh 9 stones each, value L. 2: Now, the bulls are half the weight, the cows half the value, and the hogs half the number of the whole herd: I demand the number of each? Answer, 4 bulls, 5 cows, 9 hogs.

Or any multiple of these numbers.

Prob. 29. The sum of the square and simple power of a number is 56, and the sum of the cube and square of the same is 392: The number is required? Answ. 7.

This might also be a cubic equation.

Prob. 30. The sum of the three sides of a right-angled triangle make 150, and their product, if continually multiplied, is 97500: Required the sides?

Answer, 25, 60, and 65.

Let a , e , y , be the three sides, y the hypotenuse, then $a+e+y=150$; or for the sum of the sides put s , and for their product p , then $a+e+y=s$, and $a+e=s-y$, square both sides, $a^2+2ae+e^2=s^2-2sy+y^2$. Another equation arises from the square of the hypotenuse, being equal to the sum of the squares of the base, and perpendicular; therefore $a^2+e^2=y^2$; subtract this
from

from the former equation, there remains $2ae = s^2 - 2sy$,
and $a = \frac{s^2 - 2y}{2e}$. A third equation from the given

product, is $ae y = p$, and $a = \frac{p}{ey}$; so $\frac{s^2 - 2sy}{2e} = \frac{p}{ey}$
 $s^2 ye + 2sy^2 e = 2pe$
 $y^2 - \frac{1}{2}sy = -\frac{p}{s}$
 $y - \frac{1}{4}s = \sqrt{\frac{1}{16}s^2 - \frac{p}{s}}$
 $y = 65$

Canon. Divide their product by their sum, subtract the quotient from the square of $\frac{1}{4}$ of their sum, the square root of the remainder, added to the same $\frac{1}{4}$ part, gives the hypotenuse, or longest side. It might be done otherwise, thus :

$\begin{array}{rcl} a + e & = & s - y \\ a^2 + 2ae + e^2 & = & s^2 - 2sy + y^2 \\ a^2 & & e^2 = y^2 \\ \hline 2ae & = & s^2 - 2sy \\ & & s^2 - 2ae \\ \hline y & = & 2s \end{array}$	$\begin{array}{rcl} a^2 + e^2 & = & y^2 \\ ae y & = & p \\ \hline y & = & \frac{p}{ae} \end{array}$	$\begin{array}{rcl} s^2 - 2ae & p \\ \hline & 2s \quad ae \\ s^2 ae - 2a^2 e^2 & = & 2sp \\ a^2 e^2 - s^2 ae & = & -sp \\ \hline ae - \frac{1}{4}s^2 & = & \sqrt{\frac{1}{16}s^4 - sp} \\ ae & = & 1500 \end{array}$
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Canon. From the $\frac{1}{16}$ part of the fourth power of their sum subtract their sum and product multiplied together, subtract the square root of the remainder from $\frac{1}{4}$ the square of their sum; the remainder is the product of the two legs.

Prob. 31. The sum of the sides of a right-angled triangle is 108, and the content 486: What are the sides? Answer, 27, 36, 45.

As

As before, $a + e = s - y$

$$a^2 + e^2 = y^2$$

$$a^2 + 2ae + e^2 = s^2 - 2sy + y^2$$

Subtract a^2 $e^2 =$ y^2 $ae = 972$ or p .

$$2ae = s^2 - 2sy$$

$$ae = \frac{1}{2}s^2 - sy = 972$$

$$sy = \frac{1}{2}s^2 - 972 \text{ and } y = 45$$

The last equation $ae = 972$ is had by calling the product of the legs double the content, and the rule to be drawn of this operation is thus: Square the sum of the sides, half it, and subtract from it double the content, divide the remainder by the same sum, quotes the longest. Or thus: From the square of the sum of the sides subtract quadruple the content, divide the remainder by double the sum of the sides.

Prob. 32. A long square is such, that its length is double its breadth, and its area triple its perimeter: Required the dimensions? *Answ.* 18.9.

Let its breadth be a , its length $2a$, then will its perimeter be $6a$, and its content $2a^2$, which is equal to $18a$, &c.

Prob. 33. There is an oblong whose length is 20 chains more than its breadth, and its area 12 times its perimeter: Required the dimensions?

Answer, Length 60, breadth 40 chains.

Call the breadth a , the length $a + 20$ then is the area $a^2 + 20a$, and the perimeter $4a + 40$, and $a^2 + 20a = 48a + 480$.

Prob. 34. The perimeter of a right-angled triangle is 389, and the sides are geometrical proportionals: Required the sides?

Answer, 100, 127.2, and 161.8 nearly.

Let the shortest side be a , the next ae , and the longest ay : then

M m

$a^2 +$

$$\begin{array}{rclcl}
 a^2 + a^2e^2 = a^2y^2 & a^2y = a^2e^2 & y^2 - 1 = y & a + ae + ay = 389 & \\
 e^2 = y^2 - 1 & e^2 = y & y = 1.618 & 389 & \\
 & & e = 1.272 & a = \frac{389}{1+e+y} &
 \end{array}$$

Prob. 35. Three unequal squares adjoining make one field, whose length is 19 chains, circuit 54, and content 12 acres and a half: Required a side of each square? Answer, 8, 6, and 5 chains.

From the content we have this equation, $a^2 + e^2 + y^2 = 125$; from the length, $a + e + y = 19$; and from the circuit, $4a + 2e + 2y = 54$. Subtract the first from the square of the second, thus:

$$\begin{array}{r}
 a^2 + e^2 + y^2 + 2ay + 2ae + 2ey = 361 \\
 a^2 + e^2 + y^2 = 125 \\
 \hline
 2ay + 2ae + 2ey = 236 \\
 a = \frac{118 - ey}{e + y}
 \end{array}$$

From the second equation, $a = 19 - e - y$; from the third, $a = \frac{27 - e - y}{2}$.

So we have three values of a .

$$\begin{array}{r}
 \frac{118 - ey}{e + y} = \frac{27 - e - y}{2} \\
 236 - 2ey = 27e + 27y - e^2 - 2ey - y^2 \\
 118 - ey = 19e + 19y - e^2 - 2ey - y^2 \\
 \hline
 118 - ey = 8e + 8y \\
 a = \frac{118 - 8y}{8 + y} \quad \frac{118 - ey}{e + y} = 19 - e - y \\
 118 - ey = 19e + 19y - e^2 - 2ey - y^2
 \end{array}$$

$$\frac{27-e-y}{2}=19-e-y$$

$$27-e-y=38-2e-2y$$

$$e=11-y$$

$$\frac{118-8y}{8+y}=11-y$$

$$118-8y=88+3y-y^2$$

$$y^2-11y=-30 \text{ \&c. } y=5$$

Prob. 36. In a right-angled triangle the difference of the legs is 30, and the difference of the segments, if a perpendicular is let fall from the vertex upon the hypotenuse, is 42 : Required the side?

Answer, 90, 120, 150.

Let the shortest side be a , the next $a+5d$, the least segment e , the other $e+7d$, and the perpendicular y , d being 6, and you have three equations :

1st. $2a^2+10da+25d^2=4e^2+28de+49d^2$

2d. $a^2+10da+25d^2=e^2+14de+49d^2+y^2$

$$\begin{array}{rcl} a^2 & & = 3e^2 + 14de - y^2 \\ a^2 & & = e^2 \quad + y^2 \end{array}$$

$$\begin{array}{rcl} 2a^2 & & = 4e^2 + 14de \\ a^2 = 2e^2 + 7de & & \end{array}$$

3d. $a^2=e^2+y^2$

$$\begin{array}{rcl} a^2+10da+25d^2 & = & e^2+14de+49d^2+y^2 \\ a^2 & = & e^2 \quad + y^2 \end{array}$$

$$\begin{array}{rcl} 10da+25d^2 & = & 14de+49d^2 \\ a & = & \frac{7e+12d}{5} \end{array}$$

Or squared, $a^2 = \frac{49e^2+168de+144d^2}{5}$

$$\begin{array}{rcl} \frac{49e^2+168de+144d^2}{25} & = & 2e^2+7de \\ e^2+7de & = & 144d^2 \end{array}$$

Or in numbers, $e^2+42e=5184$, and $e=54$

Prob. 37.

Prob. 37. A field of 96 acres is divided thus: There is a square in the middle, and at each end an equicrural triangle, whose base is the side of the square, the length of the whole field is 56 chains: I demand what length of dyke incloses and divides the same field? Answer, 176 chains.

Let a side of the square be a , and the perpendicular of one of the triangles e ; then $a+2e=56$, or $e = \frac{56-a}{2}$. Again, $a^2+ae=960$, and $e = \frac{960-a^2}{a}$

$$\text{And } \frac{56-a}{2} = \frac{960-a^2}{a}$$

$$56a-a^2=1920-2a^2$$

$$a^2+56a=1920$$

$$a+28=\sqrt{2704}$$

$$a=24$$

The side of the square 24 take from 56 the length, half the remainder is the perpendicular of one of the triangles, to which making half 24 a base, you find the hypotenuse, or side of the triangle to be 20; then four times a , the side of the square, and four times the same side of the triangle is the length of the dyke.

Prob. 38. Shipping a cargo of slaves on the Guinea coast for the West-Indies, 10 of them died in the voyage; however, selling them at L.40 each, I made 20 per cent. gain; shipping another cargo, of 20 more than before, 15 of them died, and raising my price to L.48, I gained 40 per cent. How many slaves in a cargo? Answer, 100 and 120.

Prob. 39. A gentleman letting a number of acres for a year, the rent amounted to L.1000; next year let 100 acres fewer, but raising the price 18d. per acre, the rent came to 1000 guineas: Required the rate of an acre? Answer, 12s. 6d.

Prob. 40.

Prob. 40. The month of the year, the day of the month, and hour of the day being added make 32, and the day of the year, number of the month, and hour of the day make 122, the hour of the day exceeds the day of the month, by the number of the month: Required month, day and hour?

Answer, April 12th, 4 afternoon.

Prob. 41. Three men depart from one place; A goes east, B west, and C south, till the distance betwixt A and B is 132 miles, betwixt A and C 25 miles, and betwixt B and C 113 miles: How far has each travelled?

Answer, A 20, B 112, C 15 miles.

Prob. 42. Three men are born; one in the year 1750, another in 1770, a third in 1784: How long must they live to make the ages of them all three geometrical proportionals Answer, 32 years, 8 months.

Prob. 43. Three equal circles that touch one another, inclose a triangular space which is just an acre: I demand the diameter of one of the circles?

Answer, 15 chains, $75\frac{1}{4}$ links.

To raise an equation here, consider, that an equilateral triangle, whose side is equal to the diameter of one of the circles, will exceed, in content, the inclosed space required, by half the content of one of the circles.

Prob. 44. Four equal circles, that touch one another, being described on the four corners of a square, inclose a kind of quadrangular space, which is an acre: Required the diameter of a circle?

Answer, 6 chains, $82\frac{1}{2}$ links nearly.

The space required is the same as the difference between a square and an inscribed circle.

Prob. 45.

Prob. 45. There are 6 inclosures adjoining to one another, containing 348 acres, 28 falls; the first four are equal, and together make a complete circle; the two last are also equal, and with two of the former, make a complete square: What is the content of each?

Answ. The first four 49 acres, 14 falls; the last two 75 acres, 3 roods, 26 falls.

Prob. 46. There are two cheefes whose solidities are equal, the one an inch broader, the other an inch thicker than its fellow, and a string of 4 feet will go round both: Required the dimensions?

Answ. $\left\{ \begin{array}{l} 16.494 \\ 15.494 \end{array} \right\}$ inches broad, $\left\{ \begin{array}{l} 7.506 \\ 8.506 \end{array} \right\}$ thick.

III. CUBIC EQUATIONS.

When an equation involves three dimensions of the unknown quantity, viz. the cube, square, and simple power, it is called a *Cubic Equation*, or though the square be wanting, and it involve only the cube, and simple power, it is still called a *Cubic*, and in that case is said to want the second term; this happens sometimes, and when otherwise, it may always be reduced, so as to want the second term, and therefore we treat of such in the first instance.

Suppose this equation $a^3 + a = 10000$ were given, and the value of a were required, we see, that if a , the unknown quantity, were deducted from 10000, there would remain just the cube of a , so that, extracting the cube root of the remainder, were it possible to be known, would resolve the equation. Now, this would be effected, if, in extracting the cube root, we always subtracted from the resolvend the same figure we put

put in the quotient; but much easier, if we add 1 to our divisor.—See the work:

$$\begin{array}{r}
 a^3 + a = 10000(21.528874940224 \&c. \\
 .8 \ 2 \\
 \begin{array}{r}
 1201 \} \text{---} \\
 123 \} 1980 \\
 \text{---} \quad \text{---} \\
 1201 \\
 61 \\
 \text{---} \\
 1262 \\
 \begin{array}{r}
 132400 \} \text{---} \\
 6375 \} 718000 \\
 \text{---} \quad \text{---} \\
 662000 \\
 15750 \\
 125 \\
 \text{---} \\
 677875 \\
 \begin{array}{r}
 13877500 \} \text{---} \\
 26052 \} 40125000 \\
 \text{---} \quad \text{---} \\
 27755000 \\
 25808 \\
 \text{---} \\
 27780808 \\
 \text{---} \\
 1390355200)12344192000
 \end{array}
 \end{array}
 \end{array}$$

Here the cube root of the first period being 2, and standing in the place of tens, subtract it from the given quantity, and at the same time its cube from the first period, and you have 1980 for a resolvend; find your divisor as usual 1200, to which add 1, and go on to extract the root as far as you chuse; but when you have got half the figures you intend, you may find the rest by contracted division, or the greater half may be found so.

As

As the most laborious part of the work in extracting the cube root when the figures are numerous, is the squaring the quotient for a new divisor, I have found a way to make that part of the work entirely easy, which I reckon a most useful contraction. The squares of any number being known, the square of that number increased by one figure, may be had thus: Multiply the increased number by its last figure, and again by the same figure, but the second time omit multiplying the right-hand figure; add these two lines; add also their sum to the former square, shifting two places to the right, and you have the square of the increased number. Now, upon this principle, to find a new divisor, in extracting the cube root, multiply your quotient twice by the last figure of it, only the second time pass the right-hand figure, and adding the two lines, triple their sum; add this to your former divisor, and annex two ciphers, and your new divisor is had with very little trouble, so in the last example, the first divisor is 1201. To find the next divisor, the quotient being 21, work thus:

21

1

—
21

2

—
41

Which 41 tripled is 123; add this to the first divisor, annexing two ciphers, you have 132400 for a second divisor. To find the next divisor, the quotient 215, do thus:

215

5

—
1075

105

—
2125

This

The triple of this 6375 added as before, makes 13877500 for your third divisor. And for the next, your quotient being 2152, as under :

$$\begin{array}{r}
 2152 \\
 2 \\
 \hline
 4304 \\
 430 \\
 \hline
 8604
 \end{array}$$

This tripled, and added to the last, gives a new divisor still. By finding the divisor this way, we also save the trouble of adding the co-efficient of the unknown quantity more than once, which would be to add to every new divisor found the common way. Proceed now to another example.

If the simple power of the unknown quantity has a co-efficient prefixed, your first divisor must be increased by that co-efficient ; and its product by the first quotient figure must be subtracted from the given quantity, thus :

$$a^3 + 5a = 60(3.49099 \text{ \&c.})$$

$$\begin{array}{r}
 42 \\
 \hline
 3200)18000 \\
 \hline
 12800 \\
 1440 \\
 64 \\
 \hline
 14304 \\
 \hline
 3696 \text{ \&c.}
 \end{array}$$

Carry on the work as far as you please, finding your divisors as taught above: the cube of 3 here was 27, to which add 3 times the co-efficient of a ,
N n
made

made 42; which we subtracted together from the given quantity. Our first divisor was 2700 to which add the same co-efficient, made 3200; for the 5 must be added here in the place of hundreds, as you may understand by considering where its product ought to be subtracted, or if the given quantity consist of one period, this figure must be added three places from the right; but if of two periods, add it to the unit figure.

Another Example.

$$a^3 + 7a = 2 \quad (.28249374 \text{ \&c.})$$

1.4

600

8

71200

1152

592000

569600

3840

512

573952

7235200

3372

18048000

14470400

3368

14473768

723857200

3574232000

28954288

6788032

6514714

273318

218157

Our first figure here was a decimal, and its product by 7, subtracted from 2, leaves .6; from this take the cube of .2, which is .008, remains .592000; then our divisor is 1200, to which prefix 7, the co-efficient. This is really putting it in the place of hundreds, if you remember your quotient figure was a decimal; then go on with extracting, and when you have gotten 3 figures, you find other 4 by Contracted Division.

But if the simple power of the unknown quantity has a negative sign, in this case you must add where you subtracted before, as it is plain the known quantity wants the product of the unknown by that figure, to make up the cube of the unknown.

Example.

$$a^3 - 5a = 1(2.3300587396 \text{ \&c.})$$

$$\begin{array}{r}
 10 \\
 \hline
 11 \\
 8 \\
 \hline
 700 \} \quad 3.000 \\
 387 \} \quad \hline
 \hline
 2100 \\
 540 \\
 27 \\
 \hline
 2667 \\
 \hline
 108700 \} \quad 333000 \\
 4167 \} \quad \hline
 \hline
 326100 \\
 6210 \\
 27 \\
 \hline
 332337 \\
 \hline
 663000 \text{ \&c.}
 \end{array}$$

Here

Here the product of the co-efficient of a , by 2, the first figure in the quote, was 10 to be added; and then subtract 8, the cube of 2, annex three ciphers as usual, and find your first divisor 1200; from which take 5, your co-efficient in the place of hundreds, as before directed, and go on with the operation, finding the divisor the contracted way; and having got 5 figures by extracting the root, you may find other 5 by Contracted Division.

Another Example.

$$a^3 - 12a = 10000 (21.73 \text{ \&c.})$$

$$\begin{array}{r}
 24 \\
 \hline
 10240 \\
 8 \\
 \hline
 1188 \} 2240 \\
 123 \} \hline
 \hline
 1188 \\
 61 \\
 \hline
 1249 \\
 \hline
 131100 \} 991000
 \end{array}$$

But supposing the equation to contain the square also, which is often the case, then that equation may be reduced to another, which will want the square, or second term, by the following rule, which is universally known.

Make choice of another unknown quantity, annex to it a third part of the co-efficient of the second term, with a different sign from what it has in the equation; make this equal to the required quantity, and substitute its powers in place of the other, and you shall have an equation wanting the second term. Let this equation

equation be given, $a^3 - 9a^2 + 30a = 112$. Here let $e+3=a$, then the cube of $e+3$ is $e^3 + 9e^2 + 27e + 27$; from which take 9 times its square, and add 30 times itself, and you have $e^3 + 3e = 76$, and this resolved, gives $e=4$, and $e+3$ was equal to a , therefore the value of a must be 7.

Another example is this, $a^3 - 15a^2 + 63a = 50$.

To exterminate the second term, make $e+5=a$.

Then,

$$\begin{array}{rcl} a^3 & = & e^3 + 15e^2 + 75e + 125 \\ -15a^2 & = & -15e^2 - 150e - 250 \\ 63a & = & 63e + 315 \\ \hline e^3 & * & -12e + 65 = 50 \text{ or } e^3 - 12e = -15 \end{array}$$

Now find the value of e , thus:

$$\begin{array}{r} -15(2.396475 \\ 24 \\ \hline .9 \\ 8 \\ \hline 00 \} 1000 \\ 387 \} \hline 0000 \\ 540 \\ 27 \\ \hline 567 \\ \hline 38700)433000 \end{array}$$

Here the known quantity being negative, 24 added, makes only 9; from which take 8, the cube of the first quotient figure, our first divisor would be 1200, but 12, ourco-efficient, being subtracted in its proper place,

place, leaves nothing, &c. Having found the value of e to be 2.396, a , being 5 more, must be 7.2396475, which is gotten with very little trouble, after a little practice. So that I reckon this method a valuable discovery, when compared with the jargon we meet with in other authors, about *Transmutations*, *Limitations*, and *Approximations*, and what brings us never the nearer our purpose.

Prob. 1. What two numbers are these, whose sum is the square of the greater, and whose difference is the square of the lesser?

Answer, 1.543688 and .839286.

$$\begin{aligned} a+e &= a^2 & a-e &= e^2 \\ a &= a^2 - e & a &= e^2 + e \quad \text{and} \quad a^2 = e^4 + 2e^3 + e^2 \end{aligned}$$

$$\begin{aligned} a^2 - e &= e^2 + e \\ a^2 &= e^2 + 2e & e^4 + 2e^3 + e^2 &= e^2 + 2e \quad \text{and} \quad e^3 + 2e^2 = 2 \end{aligned}$$

Now, to throw out the square, make $y - \frac{2}{3} = e$; then $e^3 = y^3 - 2y^2 + \frac{4}{3}y - \frac{8}{27}$, and $2e^2 = 2y^2 - \frac{8}{3}y + \frac{8}{9}$, and these added, make $y^3 - 1.33y = 1.407$; find the value of y , as under, and as before directed, from which subtract $\frac{2}{3}$, and you have the value of e . Take care to manage the repetends properly; your first divisor will be 300, from which take 1.33.

$$y^3 - 1.33y = 1.407(1.5059534$$

$$\begin{array}{r} 1\ 333 \\ \hline 2.740 \\ 1 \\ \hline 166)1740 \\ \hline 833 \\ 750 \\ 125 \\ \hline 1708 \end{array}$$

32407 &c.

Prob. 2. What two numbers are these, whose sum is equal to the quotient of the greater, divided by the lesser, and their difference equal to the quotient of the lesser, divided by the greater?

Answer, 1.191486 and .647799.

Let ae be the greater number, and a the lesser.

$$\begin{aligned} ae + a &= \frac{ae}{a} & ae - a &= \frac{a}{ae} & \frac{e}{e+1} &= \frac{1}{e^2 - e} \\ a &= \frac{e}{e+1} & a &= \frac{1}{e^2 - e} & e^3 - e^2 - e &= 1 \end{aligned}$$

To exterminate the square, make $y + \frac{1}{3} = e$, and you will find this equation $y^3 - 1.33y = 1.407$, and its value the same as in the last problem, viz. 1.5059534; to which add .333, gives the value of e , 1.8392867; and this divided by itself plus 1, gives a , that is, the least of the numbers, and a and e multiplied, gives the greater.

Prob. 3. The sum of three numbers is 30, their product 780, and the sum of their squares 338: Required the numbers? Answer, 5, 12, and 13.

$$a + e = 30 - y \qquad ae = \frac{780}{y}$$

$$\text{Squared, } \begin{array}{rcl} a^2 + 2ae + e^2 & = & 900 - 60y + y^2 \\ a^2 & & e^2 = 338 \end{array} \quad \begin{array}{rcl} & & -y^2 \\ \hline & & 281 - 30y + y^2 = \frac{780}{y} \end{array}$$

$$\text{Or, } \begin{array}{rcl} 2a^2 & = & 562 - 60y + 2y^2 \\ ae & = & 281 - 30y + y^2 \end{array} \quad \begin{array}{rcl} y^3 - 30y^2 + 281y & = & 780 \end{array}$$

$$\begin{array}{rcl} \text{Make } x + 10 = y & \text{Then, } & x^3 + 30x^2 + 300x + 1000 = y^3 \\ & & -30x^2 - 600x - 3000 = -30y^2 \\ & & + 281x + 2810 = 281y \\ \hline & & x^3 - 19x + 810 = 780 \end{array}$$

$$\text{Or, } x^3 - 19x = -30.$$

And x will be found equal to 2, 3, or -5, and therefore $y = 12, 13$ or 5.

Prob. 4. The sum of two numbers is 24, and the difference of their cubes 866: Required the numbers?
Answer, 11 and 13.

$$\begin{aligned} a + e &= 24 & e^3 &= 866 + e^3 \\ a &= 24 - e & \text{Cube this, } 13824 - 1728e + 72e^2 - e^3 &= 866 + e^3 \\ & & e^3 - 36e^2 + 864e &= 6479 \end{aligned}$$

Then making $y + 12 = e$, you have $y^3 - 432y = -433$, and $y = -1$; so is $e = 11$.

Prob. 5. Divide the number 20 into two parts, so the cube of the one part may be triple the cube of the other. Answer, 11.8108288 and 8.1891712.

$$\begin{aligned} a &= 20 - e \\ a^3 &= 8000 - 1200e + 60e^2 - e^3 = 3e^3 \\ e^3 - 15e^2 + 300e &= 2000. \text{ Make } y + 5 = e. \text{ Then,} \end{aligned}$$

$$\begin{array}{r} e^3 = y^3 + 15y^2 + 75y + 125 \\ -15e^2 = -15y^2 - 150y - 375 \\ 300e = 300y + 1500 \end{array}$$

$$y^3 + 225y + 1250 = 2000$$

$$y^3 + 225y = 750 \quad (3.1891712, \text{ or } y, \text{ and } y + 5 = e)$$

$$\begin{array}{r} 2700 \quad 75 \\ 225 \quad 27 \\ \hline 25200 \quad 48000 \\ 183 \quad \hline 25200 \\ 91 \\ \hline 25291 \end{array}$$

$$2538300)2270900$$

Prob. 6.

Prob. 6. Divide the number 20 into two parts, so as the square of the one part may be equal to the cube of the other? Answer, 14.149 and 5.85.

You will find $e^3 - e^2 + 40e = 400$. And making $y + \frac{1}{3} = e$, you will find $y^3 + 39.6y = 386.740(5.5168$

198.333

188.407

125

11466) 63407

And adding $\frac{1}{3}$ to y , you have the value of e .

Prob. 7. A son is 20 years younger than his mother, and the father is the age of them both, and the product of all their ages is 12000: Required the age of each? Answer, 10, 30, and 40.

Prob. 8. There are two cubes of stone; the solidity of both is $58\frac{1}{2}$ feet; but the side of one cube is a foot less than the other: I demand the side of each cube? Answer, $2\frac{1}{2}$ and $3\frac{1}{2}$ feet.

Cube $e+1$, is $e^3 + 3e^2 + 3e + 1$; to which add e^3 , and divide by 2, you have $e^3 + 1\frac{1}{2}e^2 + 1\frac{1}{2}e + \frac{1}{2} = 29\frac{1}{4}$; then take $a - \frac{1}{2}$, and raise it to the third power; add $1\frac{1}{2}$, its second and first powers, and you have $a^3 + .75a = 29.25$, and $a = 3$.

Prob. 9. There are three farms, containing in all, 584 acres; the number of acres in the first is the square root of those in the second, and the cube root of those in the third: I demand the acres in each farm? Answer, 8, 64, and 512.

$a^3 + a^2 + a = 584$, make $e - \frac{1}{3} = a$, you will find $e^3 + \frac{2}{3}e - \frac{7}{27} = 584$, and $e = 8.33$.

O O

Prob. 10.

Prob. 10. The sum of the three sides of an Ifoſceles, or equicrural triangle make 30, and their product 784: Required the ſides? Answer, 7.7 and 16.

$$\begin{array}{rcl} 2a + e = 30 & a^2 e = 784 & 784 \\ e = 30 - 2a & 784 & \frac{784}{a^2} = 30 - 2a \\ & e = \frac{784}{a^2} & a^3 - 15a^2 = -392 \end{array}$$

Make $y + 5 = a$, and $y^3 - 75y = -142$, and $y = 2$.

Prob. 11. A number conſiſting of two digits is ſuch, That the ſum of the ſquares of its digits with half their product, make up the number itſelf, and the difference of the ſame ſquares with half their product, make half the number: Required the number? Answer, 64.

We ſuppoſe the greateſt digit to ſtand firſt, which let be ae , the leſſer digit a ; then the number will be $10ae + a$.

$$\begin{array}{rcl} a^2 e^2 + a^2 + \frac{1}{2} a^2 e = 10ae + a & a^2 e^2 - a^2 + \frac{1}{2} a^2 e = 5ae + \frac{1}{2} a & \\ 20e + 2 & 10e + 1 & \\ a = \frac{2e^2 + 2 + e}{10e + 1} & a = \frac{2e^2 - 2 + e}{2e^2 - 2 + e} & \\ \frac{2e^2 + 2 + e}{10e + 1} & & \\ \frac{2e^2 - 2 + e}{2e^2 + 2 + e} & & \\ e^3 + .6e^2 - 2.95e = 3 & & \end{array}$$

Then, to throw out the ſquare, make $y - .2 = e$, and you have

$$y^3 - 3.07y = -.306 \quad (1.7 = y \text{ and } e = 1.5, a = 4, ae = 6)$$

$$\begin{array}{r} 3.07 \\ \hline 2764 \\ \hline 1 \\ \hline -7)1764 \\ \hline -49 \\ \hline 1470 \\ \hline 343 \\ \hline 1764 \end{array}$$

The number $-.306$ being negative, is subtracted from 3.07 : Mind Addition of Negative Quantities: Also your divisor, found the ordinary way, is 300 ; from which take 3.07 , leaves -7 negative for your divisor.

Prob. 12. What two numbers are these, whose sum, product, and difference of their squares, are all three the same number?

Answer, 1.618034 and 2.618034 .

$$\begin{aligned} ae &= a + e \\ e & \\ a &= \frac{\quad}{e-1} \end{aligned}$$

$$\begin{aligned} ae &= a^2 - e^2 \\ a^2 - e^2 & \\ a &= \frac{\quad}{e} \end{aligned}$$

$$\begin{aligned} a + e &= a^2 - e^2 \\ a &= a^2 - e^2 + e \end{aligned}$$

Square this.

$$a^2 = \frac{e^2}{e^2 - 2e + 1}$$

$$\frac{a^2 - e^2}{e} = a^2 - e^2 - e$$

$$\frac{e^2}{e^2 - 2e + 1} = \frac{e^2}{e - 1}$$

$$a^2 = \frac{e^2}{e - 1}$$

$$e^2 - 2e^2 = -1$$

And $y + \frac{2}{3} = e$ and the result will be

$$y^3 - 1.3y = -.407,407(.951$$

$$1.200$$

$$792$$

$$129$$

$$10966).63407$$

Your divisor here would be 24300 , from which take 1.33 , the co-efficient of the second term, as formerly directed.

Prob. 13. Divide 100 twice in such manner, that the major part of the first division, may be the cube of

of the minor part of the second, and the major part of the second may be triple the minor of the first?

Answ. 68.0274 and 31.9725, 95.9177 and 4.0822—

$$\begin{array}{llll} a=100-e & a=y^3 & x=100-y & x=3e \\ 100-e=y^3 & 100-y=3e & 300-3y^3=100-y & \\ e=100-y^3 & 100-y & y^3-.3y=66.6 & \\ e=\frac{\quad}{3} & & & \end{array}$$

$$66.6(4.0822$$

$$1.3$$

$$68 \cdot$$

$$64$$

$$4766)4000 \text{ \&c.}$$

Prob. 14. We have a word of four letters; the sum of the numbers of the two first is the same with that of the two last; the product of the two first is 180, and of the two last 110; and the sum of the two middlemost 37: Required the word? Answ. *Love*.

Let the three given numbers be b , c , and d . Let a represent the first letter, e the second, $d-e$ the third, y the fourth.

$$\begin{array}{llll} a+e=d-e+y & ae=b & dy-ye=c & d-2e+y=\frac{b}{e} \\ a=d-2e+y & a=\frac{b}{e} & y=\frac{c}{d-e} & y=\frac{b+2e^2-de}{e} \\ \frac{b+2e^2-de}{e}=\frac{c}{d-e} & & & \end{array}$$

$$e^3-55.5e^2+829.5e=3330$$

Make $x^3+18.5=e$ and you will find $x^3-197.25x=647.5$

$$x^3=$$

$$\begin{array}{r}
 x^3 - 197.25x = 647.5(-3.5 = x \\
 \quad \quad \quad + 591.75 \\
 \hline
 -19725 \quad + 55.75 \\
 \quad 2700 \quad - 27 \\
 \hline
 -17025 \quad) \quad 82750 \\
 \hline
 \quad \quad + 85125 \\
 \quad \quad - 2250 \\
 \quad \quad \quad - 125 \\
 \hline
 \quad \quad \quad 82750 \\
 \hline
 \quad \quad \quad (0)
 \end{array}$$

Our root here is negative, and being added to 18.5, makes 15, for the second letter of the word, viz. o. Mind how to add and subtract negative quantities, otherwise the work will appear strange.

Prob. 15. What is the side of that cube, whose solidity and surface added together, make 1600?

Answer, 10.

Prob. 16. The sum of the squares of two numbers divided by their product, quotes the lesser number, and the difference of their cubes divided by the same product, quotes the greater: Required the two numbers? Answer, 3.14789 and 2.14789.

Prob. 17. The difference of two numbers is equal to their product, and the difference of their squares equal to their sum: The numbers are required?

Answer, 1.618 and .618.

A Cubic Equation has always three roots, *positive or negative*, and so is capable of three solutions; only sometimes

sometimes two of the roots are impossible : So this equation, $a^3 - 15a^2 + 74a = 120$, has three positive roots, or solutions of the equation, viz. 4, 5, and 6 ; any of which substituted for a , will be found to answer the conditions of the equation. Again, This equation, $a^3 + 12a^2 + 47a = -60$, has three negative roots, -3 , -4 , -5 . This $a^3 - 4a^2 - 39a = 54$ has two negative and one positive root, viz. -2 , -3 , and 9. This equation, $a^3 - a^2 - 41a = -105$, has two positive and one negative root, viz. 3, 5, and -7 . This $a^3 + 5a^2 + 12a = -36$ has -4 , a negative, and two impossible roots. And this $a^3 - 5a^2 + 13a = 36$ has 4, a positive, and also two impossible roots.

THE END.

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ERRATA.

E R R A T A.

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25.	1.	5s.
26.	7.	10 cohorts, a cohort 3 manipuli
30.	37.	$850\frac{1}{2}$
32.	53.	$252\frac{3}{11}$
37.	16.	$48\frac{1}{3}$
ibid.	17.	$11\ 8\frac{5}{8}$
39.	30.	L. 333 6 8
40.	41.	50 miles
41.	46.	19 hundreds
49.	5.	69048
52.	15.	50 at Geneva
53.	8.	10 gills
54.	10.	double the value
56.	6.	a lord 37s. 6d.
ibid.	8.	A, C and D, L. 494, A's gain L. 44 13 4
64.	10.	17s. $1\frac{1}{2}$ d.
66.	19.	B 24 days
67.	22.	144 m. west longitude
70.	5.	.725 Troy
80.	16.	2304 horses
97.	2.	50 oz. each
103.	6.	L. 6 13 4
105.	16.	56s.
108.	2.	$233\frac{1}{3}$ flax
117.	4.	double the Sterling money
118.	3.	$41\frac{1}{2}$ d. per piastre.
119.	2.	L. 180 12

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